

13. Division and Decimal Principles

a) Definition of Division

Definition of Division

$a \div b = x$ means:

$$\mathbf{b \times x = a}$$

Theorem 1: Fundamental Property of Fractions

Theorem: $\mathbf{a \div b = (a \times c) \div (b \times c)}$

Proof: 设 $a \div b = x$, then $b \times x = a$ (Definition of Division)

Multiply both sides by c : $(b \times x) \times c = a \times c$

$(b \times c) \times x = a \times c$ (Associative Property of Multiplication)

therefore $x = (a \times c) \div (b \times c)$ (Definition of Division)

Thus **$a \div b = (a \times c) \div (b \times c)$ Q.E.D.**

Corollary: $\mathbf{1/10 = 10/100}$

Proof: $1 \div 10 = (1 \times 10) \div (10 \times 10) = 10 \div 100$

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Theorem 2: Division and Multiplication Commutativity

$$\text{Theorem: } a \times b \div c = (a \div c) \times b$$

$$\begin{aligned} \text{Proof: } a \times b \div c &= a \times b \times (1/c) \\ &= a \times (1/c) \times b \quad (\text{Commutative Property of Multiplication}) \\ &= (a \div c) \times b \quad \text{Q.E.D.} \end{aligned}$$

Theorem 3: Distributive Property of Division

$$\text{Theorem: } (a + b) \div c = a \div c + b \div c$$

$$\begin{aligned} \text{Proof: } (a + b) \div c &= (a + b) \times (1/c) \\ &= a \times (1/c) + b \times (1/c) \quad (\text{Distributive Property of Multiplication}) \\ &= a \div c + b \div c \quad \text{Q.E.D.} \end{aligned}$$

Theorem 4: Fraction Multiplication

$$\text{Theorem: } (1/a) \times (1/b) = 1/(a \times b)$$

$$\text{Proof: 设 } x = (1/a) \times (1/b)$$

$$\text{则 } x \times (a \times b) = (1/a) \times (1/b) \times a \times b$$

$$= (1/a \times a) \times (1/b \times b) = 1 \times 1 = 1$$

$$\text{所以 } x = 1/(a \times b) \quad \text{Q.E.D.}$$

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Theorem 5: Commutativity of Multiplication and Division

$$\text{Theorem: } a \times b \div (c \times d) = (a \div c) \times (b \div d)$$

$$\text{Proof: } a \times b \div (c \times d) = a \times b \times 1/(c \times d)$$

$$= a \times b \times (1/c) \times (1/d) \quad (\text{定理4})$$

$$= a \times (1/c) \times b \times (1/d) \quad (\text{Commutative Property of Multiplication})$$

$$= (a \div c) \times (b \div d) \quad \text{Q.E.D.}$$