

20. Fraction $\times$ Whole Number: Proof

a) Today we prove: how to multiply a whole number by a fraction

What We Want to Prove

$$A \times (B/C) = (A \times B)/C$$

Whole number $\times$ fraction = (whole $\times$ numerator) / denominator

Tools We Already Know

- ① Fraction definition: B/C is the number which, multiplied by C , equals B
- ② Key insight: $B/C = B \times (1/C)$
- ③ Associative law of multiplication: $(A \times B) \times C = A \times (B \times C)$
- ④ Lesson 17 proven: $A/C + B/C = (A+B)/C$

Intuitive Understanding

$$\begin{aligned} 3 \times 2/5 &\text{ means 3 groups of } 2/5 \text{ added together} \\ &= 2/5 + 2/5 + 2/5 \\ &= (2+2+2)/5 = 6/5 \end{aligned}$$

But we need a rigorous proof, not relying on the intuition of "adding together"

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b) Proof: $A \times (B/C) = (A \times B)/C$

Step 1 Write the fraction as **numerator $\times$ (1/denominator)**:

$$B/C = B \times (1/C)$$

Step 2 Substitute into the multiplication:

$$A \times (B/C) = A \times [B \times (1/C)]$$

Step 3 Rearrange using associative law:

$$= (A \times B) \times (1/C)$$

(associative law: $A \times (B \times X) = (A \times B) \times X$)

Step 4 Write **$(A \times B) \times (1/C)$** back in fraction form:

$$= (A \times B)/C$$

Conclusion $A \times (B/C) = (A \times B)/C$

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c) Verify Using the Proven Formula

$$3 \times 2/5 = (3 \times 2)/5 = 6/5$$

$$4 \times 3/8 = (4 \times 3)/8 = 12/8 = 3/2$$

$$2 \times 7/10 = (2 \times 7)/10 = 14/10 = 7/5$$

Practice: Calculate Using the Formula

$$\text{d)} \quad 5 \times 3/7 = (5 \times \text{ }) / 7 = \text{ } / 7$$

$$\text{e)} \quad 6 \times 4/9 = (6 \times \text{ }) / 9 = \text{ } / 9$$

$$\text{f)} \quad 3 \times 5/12 = (3 \times \text{ }) / 12 = \text{ } / 12$$