

## 14. Definition and Basic Properties of Fractions

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a) Today we answer a question: What exactly is a fraction?

### Definition of a Fraction

Define  $A/B$  defined as:

The number that, multiplied by  $B$ , equals  $A$

That is:  $B \times (A/B) = A$

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Example: Use the definition to understand  $1/2$

$1/2$  is the number that, multiplied by  $2$ , equals  $1$

That is:  $2 \times 1/2 = 1$

$1/2$  

Two  $1/2$  pieces together = 1

$$2 \times 1/2 = 1 \quad \checkmark$$

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Axioms we already know (no proof needed)

- ① Commutative law:  $x \times y = y \times x$
- ② Associative law:  $(x \times y) \times z = x \times (y \times z)$
- ③ Identity:  $x \times 1 = x$
- ④ Reciprocal:  $b \times (1/b) = 1$  (when  $b \neq 0$ )

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**b) Core property proof:  $A/B = (A \times C)/(B \times C)$**

**Step 1** Let  $x = A/B$ , by fraction definition:

$$B \times x = A$$

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**Step 2** Multiply both sides by  $C$ :

$$(B \times x) \times C = A \times C$$

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**Step 3** Rearrange using associative law:

$$(B \times C) \times x = A \times C$$

*(Associative law: first multiply  $B \times x$  then  $C$  = first multiply  $B \times C$  then multiply  $x$ )*

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**Step 4** By definition of fractions, if a number  $x$  satisfies

$$(B \times C) \times x = A \times C$$

then  $x$  is  $(A \times C)/(B \times C)$

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**Conclusion**  $A/B = (A \times C)/(B \times C)$

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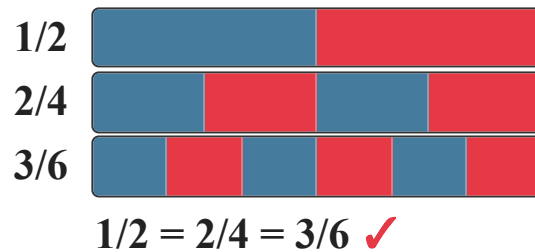
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c) Use the proven property to derive equivalent fractions

$$\frac{1}{2} = (1 \times 2) / (2 \times 2) = \frac{2}{4}$$

$$\frac{1}{2} = (1 \times 3) / (2 \times 3) = \frac{3}{6}$$

Fraction bars verify:  $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$



Practice: Complete the derivation

d)  $\frac{1}{3} = (1 \times \text{ }) / (3 \times \text{ }) = \text{ } / 6$

e)  $\frac{2}{3} = (2 \times \text{ }) / (3 \times \text{ }) = \text{ } / 6$

f)  $\frac{3}{4} = (3 \times \text{ }) / (4 \times \text{ }) = \text{ } / 8$