1. Ratio Concept

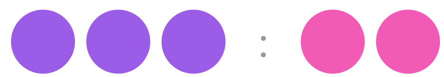
Mathematical Definition: What is a Ratio?

Ratio: A comparison of two quantities by division, showing how many times one value contains the other.

$$a : b \quad \text{or} \quad a / b \quad (b \neq 0)$$

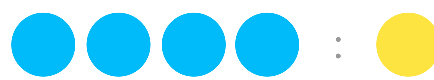
a is the first term, b is the second term

a)



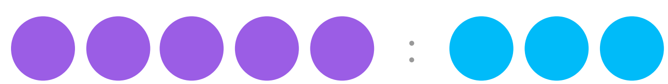
Purple : Pink = **3 : 2**

b)



Blue : Yellow = ________

c)



Purple : Blue = ________

1. Ratio Concept

d)

A bouquet has 6 red flowers and 4 yellow flowers.

Red : Yellow = ____________

Yellow : Red = ____________

e)

A class has 12 boys and 8 girls.

Boys : Girls = ____________

Girls : Boys = ____________

f)

A fruit basket has 9 apples and 6 oranges.

Apples : Oranges = ____________

Oranges : Apples = ____________

If 3 more apples are added, what is the new ratio?

Apples : Oranges = ____________

2. Unit Ratio

Mathematical Definition: What is a Unit Ratio?

A ratio a:b can be scaled up or down — **multiplying or dividing both terms by the same number keeps the ratio unchanged**.

$$a : b = (a \times c) : (b \times c) = (a \div c) : (b \div c)$$

When the second term is reduced to **1**, the first term becomes the **unit ratio**.

$$a : b = (a \div b) : 1 = a \div b \text{ (per 1 unit)}$$

a)

3 apples cost 6 yuan. How much does each apple cost?

Ratio = 6 yuan : 3 apples

▼ $\div 3$

Unit Ratio = $\boxed{2}$ yuan per apple

b)

4 books cost 20 yuan. How much does each book cost?

Ratio = 20 yuan : 4 books

▼ $\div 4$

Unit Ratio = ________ yuan per book

2. Unit Ratio

c)

5 kg of rice costs 25 yuan. How much per kilogram?

Ratio = 25 yuan : 5 kg

▼ ÷ 5

Unit Ratio = ________ yuan/kg

d)

A car travels 120 km in 2 hours. What is its speed?

Ratio = 120 km : 2 hours

▼ ÷ 2

Unit Ratio = ________ km/h

e)

6 cartons of milk cost 18 yuan. How much per carton?

Unit Ratio = ________ yuan per carton

f)

Xiaoming runs 600 meters in 3 minutes. How many meters per minute?

Speed = ________ m/min

At this speed, how far can he run in 5 minutes?

Distance = ________ m

3. Equivalence Ratio Table

Key Insight: The Unit Ratio Stays the Same

In each column, **Purple : Pink = 2 : 1**. The **unit ratio = 2**. It never changes!

a)

Purple	2	→	4	→	6	→	8
Pink	1	→	2	→	3	→	4
Unit Ratio	2	→	2	→	2	→	2

b)

Blue	4	→		→		→	
Yellow	1	→		→		→	
Unit Ratio	4	→	4	→	4	→	4

3. Equivalence Ratio Table

c)

Purple	3			
Blue	1			
Unit Ratio	3	3	3	3

d)

Pink	6			
Yellow	2			
Unit Ratio	3	3	3	3

3. Equivalence Ratio Table

e)

Blue	5	→		→		→	
Purple	1	→		→		→	
Unit Ratio	5	→	5	→	5	→	5

f)

Pink	8	→		→		→	
Blue	2	→		→		→	
Unit Ratio	4	→	4	→	4	→	4

4. Unit Ratio Word Problems

Applying Unit Ratios to Real-World Problems

With the **unit ratio**, you can find the total (**forward**) or find the quantity (**backward**).

a) 1 pen costs 3 yuan. How much for 5 pens? How many pens for 24 yuan?

Cost (yuan)	3	→	15	→	24
Pens	1	→	5	→	8
Unit Price	3	→	3	→	3

b) Apples cost 5 yuan/kg. How much for 3 kg? How many kg for 35 yuan?

Cost (yuan)	5	→		→	35
Apples (kg)	1	→	3	→	
Unit Price	5	→	5	→	5

4. Unit Ratio Word Problems

c) A car travels at 60 km/h. How far in 4 hours? How many hours for 300 km?

Distance (km)	60	→		→	300
Time (h)	1	→	4	→	
Speed	60	→	60	→	60

d) Reading 8 pages per day. How many pages in 6 days? How many days for 56 pages?

Pages	8	→		→	56
Days	1	→	6	→	
Rate	8	→	8	→	8

4. Unit Ratio Word Problems

e) Milk costs 4 yuan per carton. How much for 6 cartons? How many cartons for 28 yuan?

Cost (yuan)	4	→		→	28
Milk (cartons)	1	→	6	→	
Unit Price	4	→	4	→	4

f) Running at 200 m/min. How far in 5 minutes? How many minutes for 1800 m?

Distance (m)	200	→		→	1800
Time (min)	1	→	5	→	
Speed	200	→	200	→	200

5. Introduction to Percentages

Mathematical Definition: Percentage

A ratio a:b = a/b. When the second term is reduced to 1, you get the unit ratio. When the second term is scaled to 100, you get the percentage:

$$\text{Unit Ratio} = a \div b \quad \longrightarrow \quad \text{Percentage} = (a \div b) \times 100\,\%$$

a) Made 3 shots out of 5 attempts. How many in 25 attempts? 9 makes in how many attempts?

Makes	3	→	15	→	9
Attempts	5	→	25	→	15
Rate	0.6	→	0.6	→	0.6
Per 100	60	→	60	→	60
Percentage	60 %	→	60 %	→	60 %

b) Got 18 correct out of 20. How many correct in 100? 36 correct in how many total?

Correct	18	→		→	36
Total	20	→	100	→	
Rate	0.9	→	0.9	→	0.9
Per 100	90	→	90	→	90
Percentage	90 %	→	90 %	→	90 %

5. Introduction to Percentages

c) Attended 23 days out of 25. How many in 100 days? 46 days in how many total?

Attended	23	→		→	46
Total Days	25	→	100	→	
Rate	0.92	→	0.92	→	0.92
Per 100	92	→	92	→	92
Percentage	92 %	→	92 %	→	92 %

d) 36 items passed out of 40. How many passed in 200? 72 passed in how many total?

Passed	36	→		→	72
Total Items	40	→	200	→	
Rate	0.9	→	0.9	→	0.9
Per 100	90	→	90	→	90
Percentage	90 %	→	90 %	→	90 %

e) 17 trees survived out of 20. How many survive in 100? 34 survive in how many total?

Survived	17	→		→	34
Total Trees	20	→	100	→	
Rate	0.85	→	0.85	→	0.85
Per 100	85	→	85	→	85
Percentage	85%	→	85%	→	85%

f) Hit 15 targets out of 20. How many hits in 100? 30 hits in how many total?

Hits	15	→		→	30
Total	20	→	100	→	
Rate	0.75	→	0.75	→	0.75
Per 100	75	→	75	→	75
Percentage	75%	→	75%	→	75%

6. Ratio and Unit Conversion

Key Idea: Using Conversion Rates as Ratios

If **1 large unit = conversion rate × small unit**, you can find the total of small units (forward) or the number of large units (backward).

a) 1 m = 100 cm. 5 m = ? cm. 300 cm = ? m

Meters	1	→	5	→	3
Centimeters	100	→	500	→	300
Rate	100	→	100	→	100

b) 1 kg = 1000 g. 4 kg = ? g. 7000 g = ? kg

Kilograms	1	→		→	7
Grams	1000	→	4000	→	
Rate	1000	→	1000	→	1000

6. Ratio and Unit Conversion

c) 1 hour = 60 min. 3 hours = ? min. 240 min = ? hours

Hours	1	→		→	4
Minutes	60	→	180	→	
Rate	60	→	60	→	60

d) 1 L = 1000 mL. 2 L = ? mL. 5000 mL = ? L

Liters	1	→		→	5
Milliliters	1000	→	2000	→	
Rate	1000	→	1000	→	1000

e) 1 ft = 12 in. 4 ft = ? in. 60 in = ? ft

Feet	1	→		→	5
Inches	12	→	48	→	
Rate	12	→	12	→	12

f) 1 yd = 3 ft. 6 yd = ? ft. 21 ft = ? yd

Yards	1	→		→	7
Feet	3	→	18	→	
Rate	3	→	3	→	3

7. Fraction Divided by Fraction

a/b ÷ c/d = a/b × d/c (dividing by a fraction = multiplying by its reciprocal)

Or use **common denominator method**: rewrite with a common denominator, then divide the numerators.

a) 1/2 ÷ 1/4 = ?

Reciprocal: 1/2 × 4/1 = 4/2 = 2

Common denom: 2/4 ÷ 1/4 = 2÷1 = 2

b) 3/4 ÷ 1/4 = ?

7. Fraction Divided by Fraction

c) $1/3 \div 1/6 =$?

d) $2/5 \div 1/5 =$?

7. Fraction Divided by Fraction

e) $3/8 \div 1/4 = \ ?$

f) $5/6 \div 1/3 = \ ?$

8. Multi-Digit Division

Splitting Method

Break the dividend into multiples of the divisor, divide each part, then add or subtract the results.

a) 144 ÷ 12

$$144 = 120 + 24$$

$$120 \div 12 = 10, \quad 24 \div 12 = 2$$

$$\mathbf{144} \div 12 = 12$$

b) 5040 ÷ 14

$$5040 = 4200 + 840$$

$$4200 \div 14 = 300, \quad 840 \div 14 = 60$$

$$\mathbf{5040} \div 14 = 360$$

c) 1287 ÷ 13

$$1287 = 1300 - 13$$

$$1300 \div 13 = 100, \quad 13 \div 13 = 1$$

$$\mathbf{1287} \div 13 = 99$$

8. Multi-Digit Division

d) $7200 \div 24 =$ ______

e) $999 \div 12 =$ ______

f) $3333 \div 25 =$ ______

8. Multi-Digit Division

g) $4800 \div 25 =$ ☐

h) $2500 \div 12 =$ ☐

9. Decimal Operations

Splitting Method

Split a decimal into its integer and fractional parts, compute each part, then combine.

a) Addition: 3.5 + 2.7

$$3.5 + 2.7 = 3.5 + 2.5 + 0.2$$
$$= 6 + 0.2$$

$$\mathbf{3.5 + 2.7 = 6.2}$$

b) Subtraction: 12.4 − 5.8

$$12.4 - 5.8 = 12.4 - (5.4 + 0.4)$$
$$= 12.4 - 5.4 - 0.4 = 7 - 0.4$$

$$\mathbf{12.4 - 5.8 = 6.6}$$

c) Multiplication: 4.6 × 3

$$4.6 \times 3 = (4 + 0.6) \times 3$$
$$= 4 \times 3 + 0.6 \times 3 = 12 + 1.8$$

$$\mathbf{4.6 \times 3 = 13.8}$$

d) Division: 8.4 ÷ 2

$$8.4 \div 2 = (8 + 0.4) \div 2$$
$$= 8 \div 2 + 0.4 \div 2 = 4 + 0.2$$

$$\mathbf{8.4 \div 2 = 4.2}$$

9. Decimal Operations

e) $5.7 + 3.6 =$ ☐

f) $15.3 - 7.8 =$ ☐

g) $3.2 \times 5 =$ ☐

h) $9.6 \div 3 =$ ☐

10. GCF and LCM

Greatest Common Factor (GCF) & Least Common Multiple (LCM)

GCF = the largest factor shared by two numbers **LCM** = the smallest multiple shared by two numbers

a) Find the GCF and LCM of 12 and 18

Factors of 12: 1, 2, 3, 4, **6**, 12

Factors of 18: 1, 2, 3, **6**, 9, 18

GCF(12,18) = **6**

Multiples of 12: 12, 24, **36**, 48, 60, 72...

Multiples of 18: 18, **36**, 54, 72...

LCM(12,18) = **36**

b) Find the GCF and LCM of 8 and 12

Factors of 8:

Factors of 12:

GCF(8,12) =

Multiples of 8:

Multiples of 12:

LCM(8,12) =

10. GCF and LCM

c) Find the GCF and LCM of 15 and 25

Factors of 15:

Factors of 25:

GCF(15,25) =

Multiples of 15:

Multiples of 25:

LCM(15,25) =

d) Find the GCF and LCM of 6 and 8

Factors of 6:

Factors of 8:

GCF(6,8) =

Multiples of 6:

Multiples of 8:

LCM(6,8) =

10. GCF and LCM

e) Find the GCF and LCM of 9 and 12

Factors of 9:

Factors of 12:

GCF(9,12) =

Multiples of 9:

Multiples of 12:

LCM(9,12) =

f) Find the GCF and LCM of 24 and 36

Factors of 24:

Factors of 36:

GCF(24,36) =

Multiples of 24:

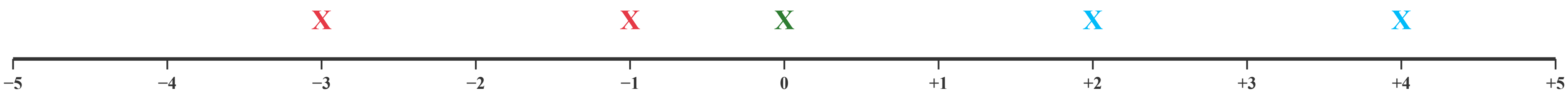
Multiples of 36:

LCM(24,36) =

11. Positive and Negative Numbers

a)

−3, +2, 0, −1, +4



Positive: **Negative:**

b)

−2, +3, 0, −4, +1, −1, +2

Positive: **Negative:**

11. Positive and Negative Numbers

c)

+50, −20, 0, +30, −10

Positive: **Negative:**

d)

+5000, −2000, 0, +3000, −1000

Positive: **Negative:**

11. Positive and Negative Numbers

e)

−6, +2, +8, −12, 0, +4

Positive: **Negative:**

f)

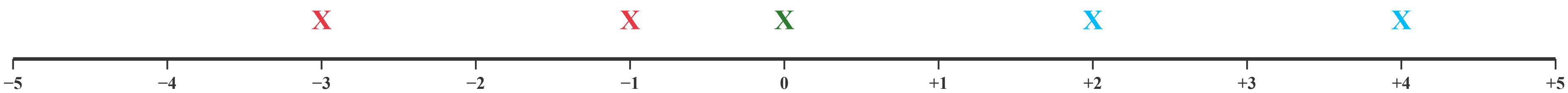
−8, +3, −2, 0, +6, −5

Positive: **Negative:**

12. Rational Numbers and the Number Line

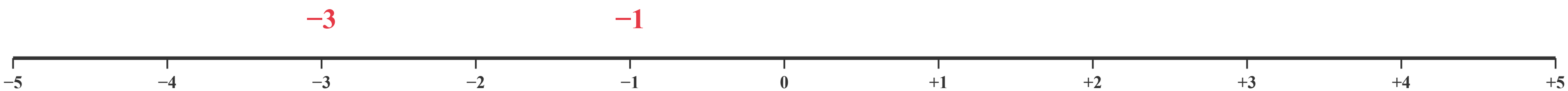
a)

−3, 0, +2, −1, +4



b)

Compare **−3** and **−1**

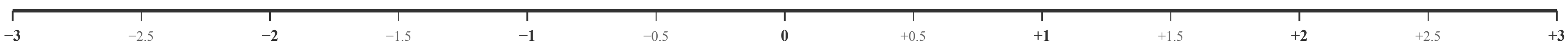


Which is larger? ______ **Why?** ______

12. Rational Numbers and the Number Line

c)

−1.5, −0.5, +1.5, +2.5



d)

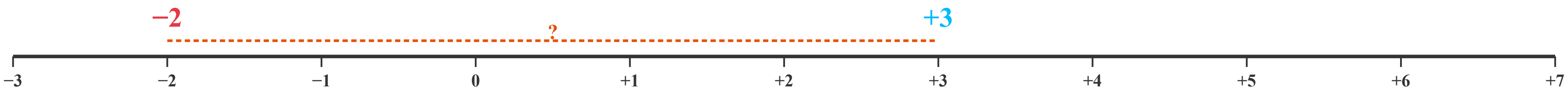
−12, −4, +3, +8

Order from least to greatest:

12. Rational Numbers and the Number Line

e)

How many units apart are **+3** and **−2** on the number line?



Distance: ______ **units**

f)

−2.5, **+1**, **0**, **−0.5**, **+2.5**

Positive: ______ **Negative:** ______

13. Ordering Rational Numbers

a)

−5, −2, 0, +3, +4

Order from least to greatest:

b)

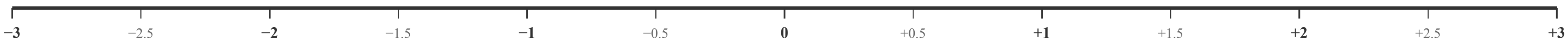
−7, −1, +2, −4

Order from least to greatest:

13. Ordering Rational Numbers

c)

−2.5, −0.5, +1.5, +2.5



Order from least to greatest:

d)

−8, +3, −1, 0, +6, −5

Order from least to greatest:

13. Ordering Rational Numbers

e)

−3.5, +1, −2, +2.5, −0.5

Order from least to greatest: ______

f)

Fill in with < or >: **−4** ☐ **−7** **0** ☐ **−2** **+3** ☐ **−5**

−9 ☐ **−6** **+5** ☐ **−1** **−8** ☐ **0**

14. Absolute Value

What is Absolute Value?

Definition

The absolute value of a number is its distance from 0

Written as |x|, read as "the absolute value of x"

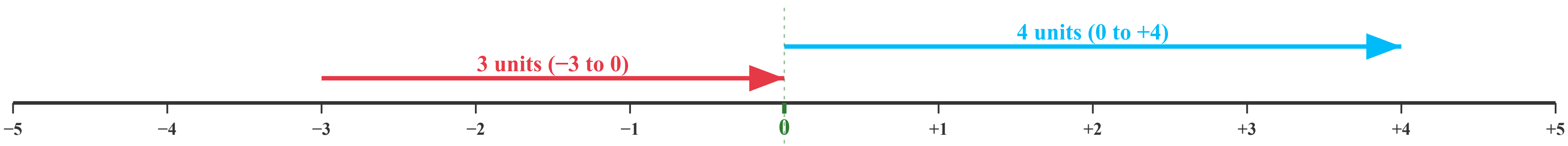
Example: |−3| = 3, because the distance from −3 to 0 is 3 units

|+4| = 4, because the distance from +4 to 0 is 4 units

---

a) Example

$|-3| = $ ______ $|+4| = $ ______



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b)

$|-5| = $ ______ $|+2| = $ ______

Which has a greater absolute value? ______

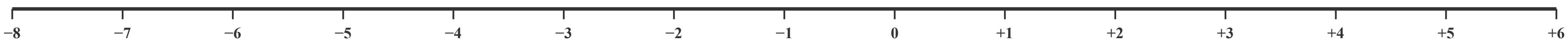
14. Absolute Value

c)

$|-6| = \square$ $\quad |0| = \square$ $\quad |+3| = \square$

d)

$\mathbf{-7} \ \square \ \mathbf{+2} \qquad |-7| \ \square \ |+2|$



e)

-9		$+4$	$\lvert -9 \rvert$		$\lvert +4 \rvert$
-5		-1	$\lvert -5 \rvert$		$\lvert -1 \rvert$
$+6$		-3	$\lvert +6 \rvert$		$\lvert -3 \rvert$

f)

Which is closest to 0? **−8**, **+5**, **−2**, **0**, **+7**

Answer:

Which is farthest from 0?

15. The Coordinate Plane

Coordinates (x, y)

x is the position on the X-axis | y is the position on the Y-axis

Find the corresponding positions on the axes, draw perpendicular lines, and the intersection is the point

a) Example: Plotting Points

Plot the following four points:

A_0 (−3, 0) A_1 (−3, 2)

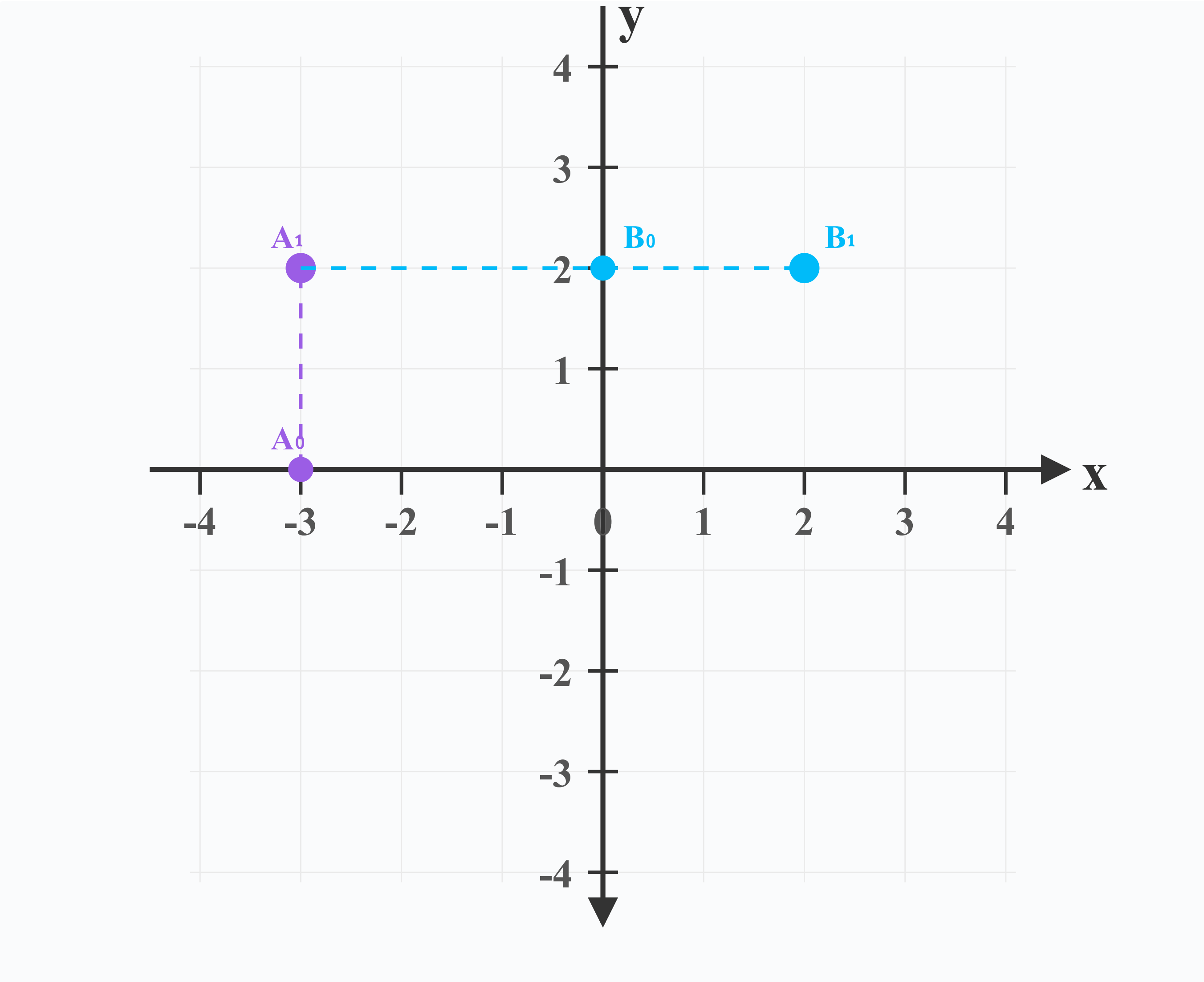
B_0 (0, 2) B_1 (2, 2)

Calculate:

Distance from A_1 to B_0 =

Distance from A_0 to B_1 =

Distance from A_1 to B_1 =



b) Points and Distances

Plot the following four points:

C_0 (−4, 0) **C_1 (−4, −1)**

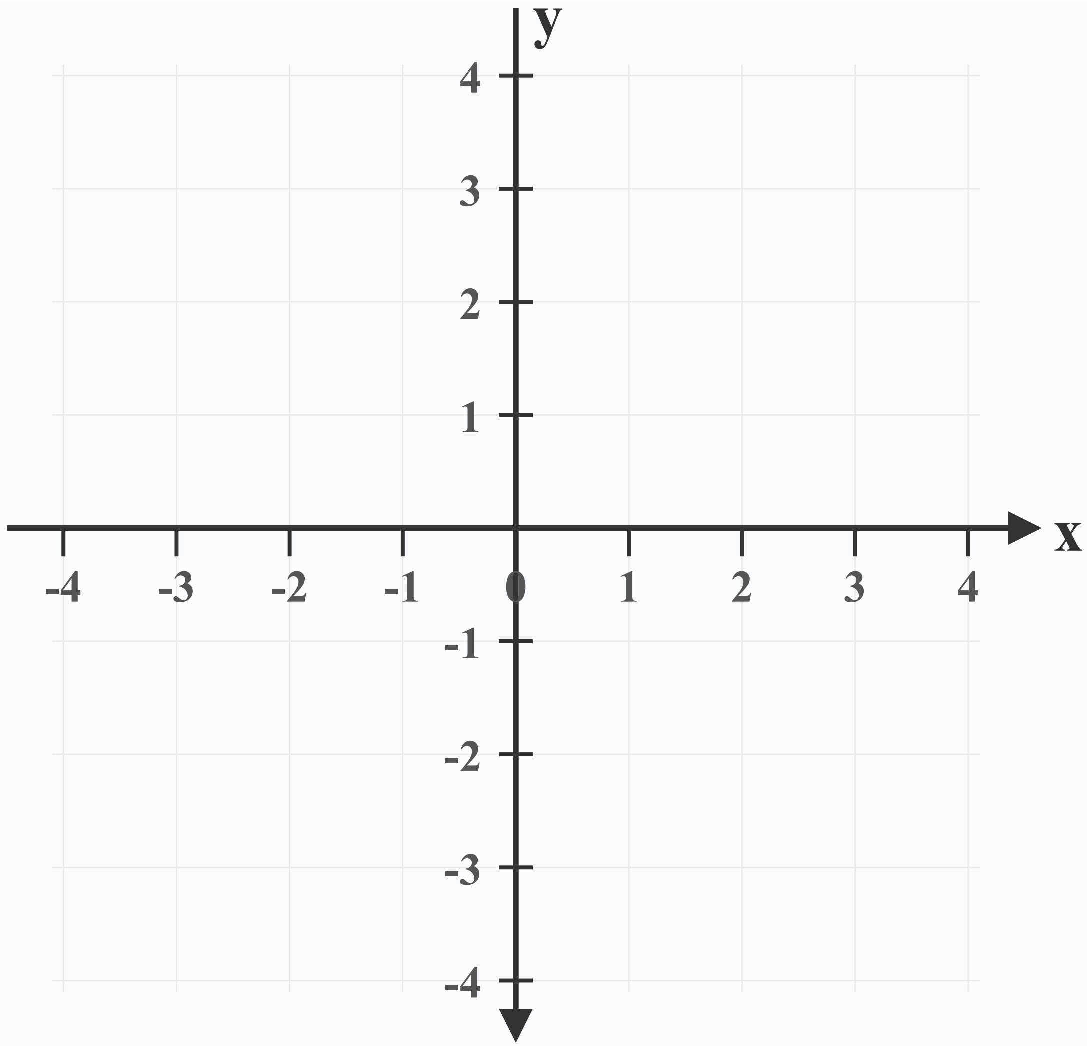
D_0 (0, −1) **D_1 (2, −1)**

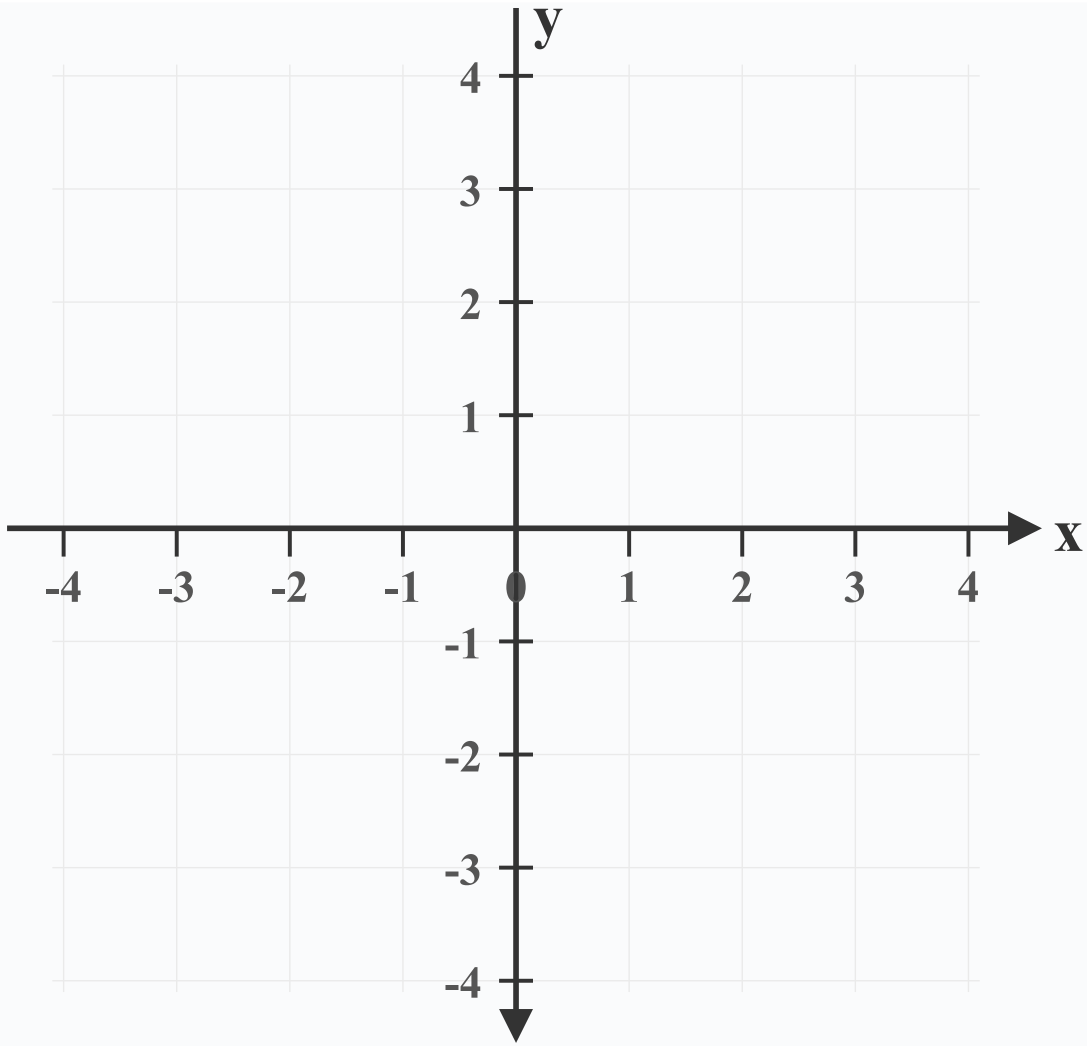
Calculate:

Distance from C_1 to D_0 =

Distance from C_0 to D_1 =

Distance from C_1 to D_1 =





c) Points and Distances

Plot the following four points:

E_0 (−2, 0) E_1 (−2, 3)

F_0 (0, −2) F_1 (−2, −2)

Calculate:

Distance from E_1 to F_0 =

Distance from E_0 to F_1 =

Distance from E_1 to F_1 =

16. Exponents and Powers

$$A^n = A \times A \times A \times \ldots \times A$$

A is the **base**, n is the **exponent**, and the result is called the **power**

a)

$$3 \times 3 \times 3 = 3^3 = 27$$
$$5 \times 5 \times 5 \times 5 = 5^4 = 625$$

b)

$$4 \times 4 = \underline{\qquad}$$
$$2 \times 2 \times 2 \times 2 \times 2 = \underline{\qquad}$$

c)

$3^2 = \underline{\quad}$ $10^3 = \underline{\quad}$

$2^6 = \underline{\quad}$ $1^5 = \underline{\quad}$

d)

3^2 ☐ 2^3

4^2 ☐ 2^4

5^2 ☐ 2^5

e)

The side length of a square is 6. What is its area? (Express in exponential form and calculate)

Area = ☐

f)

The side length of a cube is 4. What is its volume? (Express in exponential form and calculate)

Volume = ☐

17. Variable Expressions

What is a Variable?

Variable: A letter (such as x, y, n) that represents an unknown or changing number

Expression: A mathematical phrase that connects numbers and variables using operation symbols

Examples: $x + 5$, $2n - 3$, $4y$

a)

Example: Evaluating Expressions

When $x = 3$, $x + 5 =$ ___

When $n = 7$, $2n - 3 =$ ___

b)

When $x = 4$, evaluate the following expressions

$x + 9 =$ ___ $3x =$ ___

$x - 2 =$ ___ $2x + 1 =$ ___

c)

When $y = 6$, evaluate the following expressions

$y + 7 =$ ___ $4y =$ ___

$y - 5 =$ ___ $3y - 8 =$ ___

17. Variable Expressions

d)

When n = 10, evaluate the following expressions

$n + 15 =$ ______ **$n - 3 =$** ______

$2n =$ ______ **$n \div 2 =$** ______

e)

When a = 5, b = 3, evaluate the following expressions

$a + b =$ ______ **$a - b =$** ______

$2a + b =$ ______ **$a \times b =$** ______

f)

Write Expressions

A number 7 greater than x: ______

3 times n: ______

A number 4 less than y: ______

18. Equivalent Expressions

What are Equivalent Expressions?

Equivalent Expressions: Two expressions that are identical when expanded using properties of operations

Key Property: Distributive Property $a(b+c) = ab + ac$

Commutative Property $a+b=b+a$ | Associative Property $(a+b)+c=a+(b+c)$

a)

Example: Expand and Determine Equivalence

$$3(x + 2) = 3x + 3\times2 = 3x + 6 \rightarrow \text{Equivalent ✓}$$

$$2(x + 3) = 2x + 6 \neq 2x + 3 \rightarrow \text{Not Equivalent ✗}$$

b)

Expand and determine if equivalent (fill ✓ or ✗)

$4(x + 1)$ $4x + 4$ ___

$2(x + 3)$ $2x + 3$ ___

$5(2 + x)$ $10 + 5x$ ___

c)

Expand the following expressions

$3(x + 5) =$ ___

$2(4 + y) =$ ___

$6(n - 2) =$ ___

18. Equivalent Expressions

d)

Combine Like Terms

$3x + 2x =$

$5y + y - 2y =$

$4n + 3 + 2n + 1 =$

e)

Expand and determine equivalence

$2x + 3x \quad 5x$

$4y - y \quad 3$

$3(n + 2) \quad 3n + 6$

f)

Write an equivalent expression for each

$2x + 4x =$

$3(x + 1) =$

$5 + 2y + 1 + 3y =$

19. Introduction to Equations

What is an Equation?

Equation: A mathematical sentence that contains an unknown, such as $x + 5 = 12$

Solve an equation: Find the value of the unknown that makes the equation true

Method 1: Substitution Method (Trial Method) — Try substituting different numbers to find which one makes the equation true

a)

Example: Solving equations using substitution

$x + 5 = 12$ → Try x=6: 6+5=11✗ Try x=7: 7+5=12✓ → $x = 7$

$2y = 14$ → Try y=6: 2×6=12✗ Try y=7: 2×7=14✓ → $y = 7$

b)

Use substitution to verify which is the solution of the equation

$x + 4 = 9$ $x = 5$ ☐ $x = 4$ ☐

$y - 3 = 8$ $y = 10$ ☐ $y = 11$ ☐

c)

Solve the equation using substitution

$x + 6 = 15$ → $x =$ ☐

$y - 4 = 9$ → $y =$ ☐

$3n = 18$ → $n =$ ☐

d)

Solve the equation

$x + 8 = 20 \rightarrow x =$ ____

$y - 7 = 5 \rightarrow y =$ ____

$4z = 24 \rightarrow z =$ ____

e)

Solve the equation

$x + 12 = 30 \rightarrow x =$ ____

$y - 9 = 11 \rightarrow y =$ ____

$6n = 42 \rightarrow n =$ ____

f)

Write and solve equations based on the problem

A number plus 15 equals 28, what is this number?

Equation: ____ **Answer:** ____

5 times a number equals 35, what is this number?

Equation: ____ **Answer:** ____

20. Variables and Real-World Problems

Variables Describe the Real World

Use a variable to represent an unknown → Write an expression → Substitute and evaluate

a)

A box contains n pencils. Buy 3 boxes.

Expression **$3n$**

Substitute **When $n = 12 \rightarrow 3 \times 12 = 36$ pencils**

b)

Tom is x years old. His mom is 25 years older than Tom.

Expression ______

Substitute **When $x =$ ____ → Mom is ____ years old**

c)

An apple costs y dollars. Buy 5 apples.

Expression ______

Substitute **When $y =$ ____ → Total ____ dollars**

20. Variables and Real-World Problems

d)

A rectangle has length a and width b.

Perimeter ____

Area ____

Substitute **When a = ____, b = ____ → Area = ____**

e)

Store sale: Original price p dollars, 20% off.

Expression ____

Substitute **When p = ____ → Sale price ____ dollars**

Substitute **Buy 3 items, total ____ dollars**

f)

You are ____ years old. Your dad is ____ years older than you.

Expression **Dad's age = ____**

Substitute **You are ____, dad is ____ years older → Dad is ____ years old**

21. Solving One-Step Equations

Solve the equation: find the value of x that makes the equation true

$$x + p = q \;\rightarrow\; x = q - p$$

$$p \cdot x = q \;\rightarrow\; x = q \div p$$

a)

$$x + 5 = 12 \;\rightarrow\; x = 7 \;\; (7+5=12✓)$$
$$3x = 18 \;\rightarrow\; x = 6 \;\; (3\times6=18✓)$$

b)

$$x + 8 = 15 \;\rightarrow\; x = \_\_\_$$
$$x + 12 = 20 \;\rightarrow\; x = \_\_\_$$

c)

$$4x = 20 \;\rightarrow\; x = \_\_\_$$
$$6x = 30 \;\rightarrow\; x = \_\_\_$$

21. Solving One-Step Equations

d)

$x + 7 = 16 \;\rightarrow\; x =$

$5x = 35 \;\rightarrow\; x =$

e)

$x + 9 = 21 \;\rightarrow\; x =$

$7x = 42 \;\rightarrow\; x =$

f)

$x + 15 = 32 \;\rightarrow\; x =$

$9x = 54 \;\rightarrow\; x =$

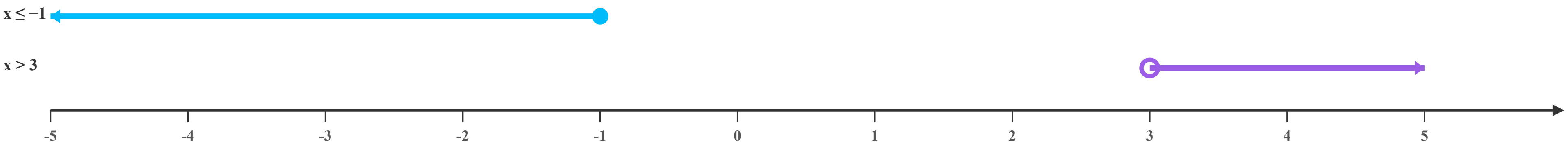
22. Inequalities

$>$ greater than $<$ less than $\geq$ greater than or equal to $\leq$ less than or equal to

Open circle → does not include this number | Closed circle → includes this number

a)

Represent the inequality on the number line

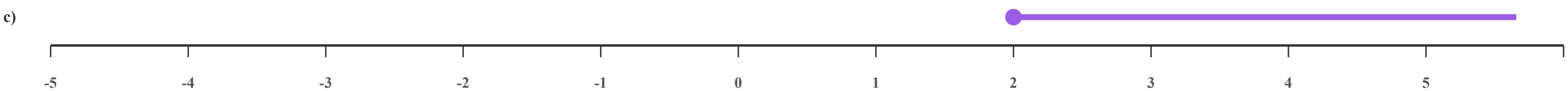


b)

Represent on the number line: $x \geq -2 \quad x < 4$

c)

Write the inequality represented on the number line



Inequality:

22. Inequalities

d)

Represent on the number line: $x < -1 \quad x \geq 3$

e)

Write the inequality represented on the number line

e)

Inequality:

f)

Write the inequality represented on the number line

f)

Inequality:

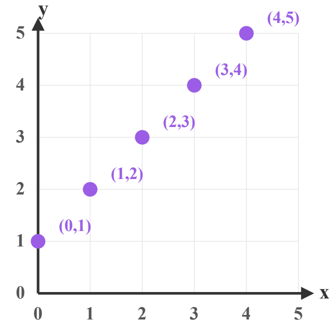
23. Independent and Dependent Variables

Independent variable x varies freely → **Dependent variable** y changes with x

Write the equation → Fill in the table → Plot points on the graph

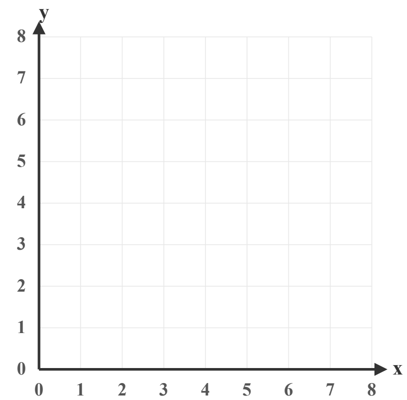
a) $y = x + 1$ Fill in the table and plot the points

x	0	1	2	3	4
y	1	2	3	4	5



b) $y = 2x$ Fill in the table and plot the points

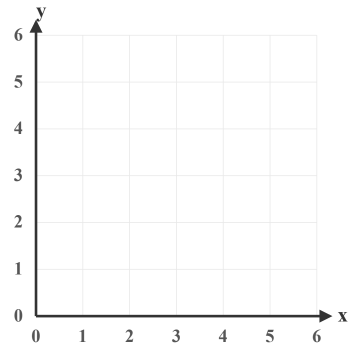
x	0	1	2	3	4
y	0	2			



23. Independent and Dependent Variables

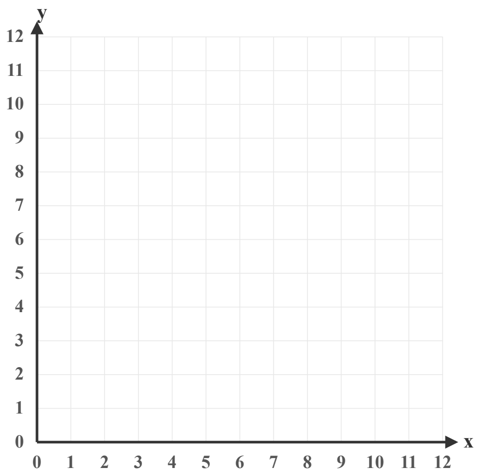
c) $y = x + 2$ Fill in the table and plot the points

x	0	1	2	3	4
y	2	3			



d) $y = 3x$ Fill in the table and plot the points

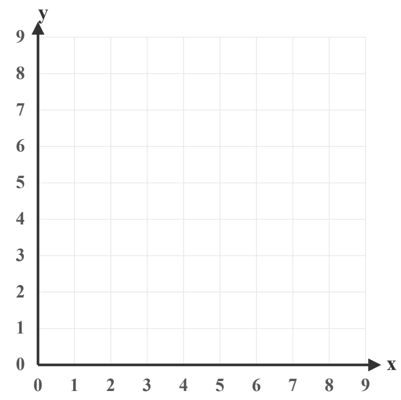
x	0	1	2	3	4
y	0	3			



23. Independent and Dependent Variables

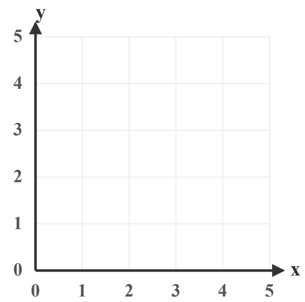
e) **$y = 2x + 1$** Fill in the table and plot the points

x	0	1	2	3	4
y	1	3			



f) **$y = x - 1$** Fill in the table and plot the points (x starts from 1)

x	1	2	3	4	5
y	0	1			



24. Area of Triangles

1 Congruent Triangles

Definition: Same shape and size, can be perfectly superimposed. Written as $\triangle ABC \cong \triangle DEF$

Conditions for Congruence

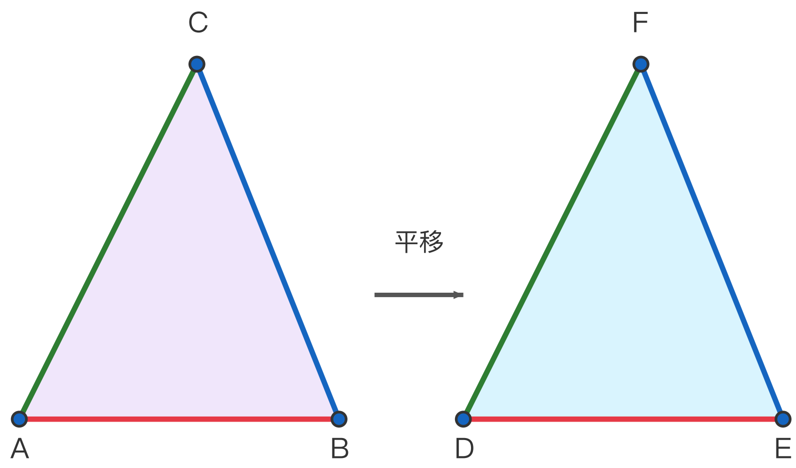
SSS — Three corresponding sides equal

SAS — Two sides and their included angle equal

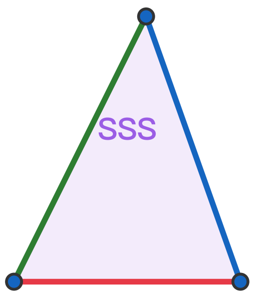
ASA — Two angles and their included side equal

AAS — Two angles and a non-included side equal (two angles equal implies the third angle is equal, equivalent to ASA)

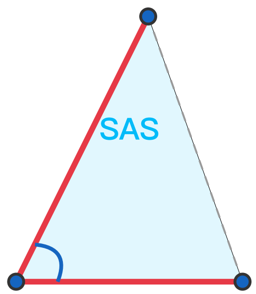
Properties: Corresponding sides are equal, corresponding angles are equal



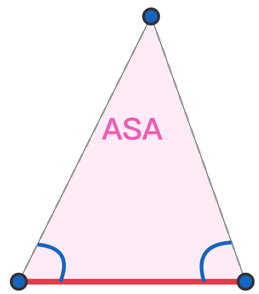
SSS



SAS



ASA



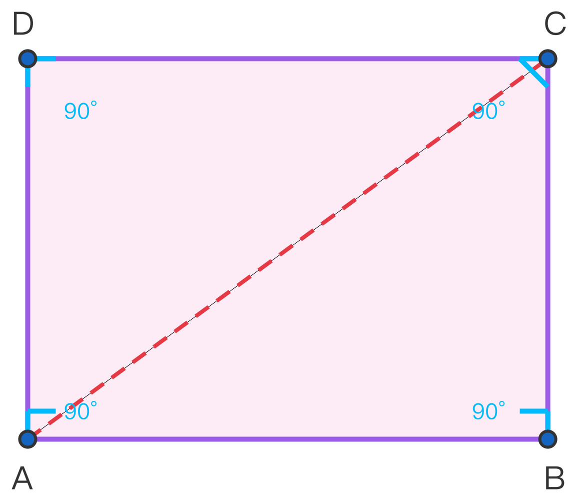
2 Rectangle

Definition: A quadrilateral with four right angles, $\angle A = \angle B = \angle C = \angle D = 90°$

Theorem: A diagonal divides a rectangle into two congruent triangles

> In $\triangle ABC$ and $\triangle CDA$:
> ① $AC = AC$ (common side)
> ② $AB = CD$, $AD = BC$ (rectangle opposite sides equal)
> ③ $\therefore\ \triangle ABC \cong \triangle CDA$ (SSS)
> $\therefore$ A rectangle is divided by its diagonal into two congruent right triangles

Area: Area of rectangle = **length $\times$ width** (fundamental fact)

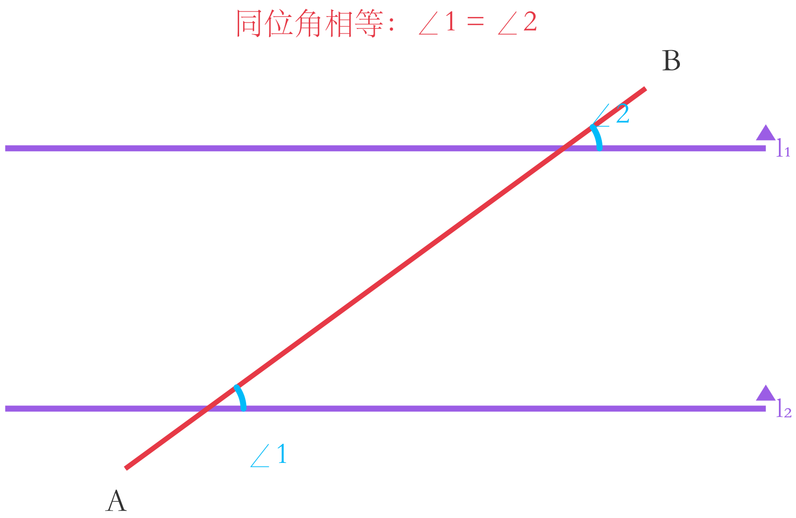


3 Corresponding Angles

Theorem: When two parallel lines are cut by a transversal, corresponding angles are equal

> As shown, $l_1 \parallel l_2$, AB is the transversal
> $\angle 1$ and $\angle 2$ are in corresponding positions, called corresponding angles
> By the parallel lines property: $\angle 1 = \angle 2$

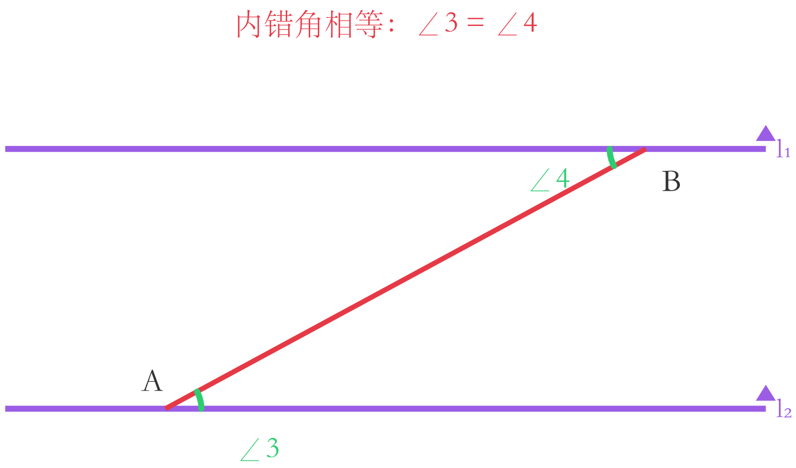
We will use this theorem to prove triangle congruence later



4 Alternate Interior Angles

Theorem: When two parallel lines are cut by a transversal, alternate interior angles are equal

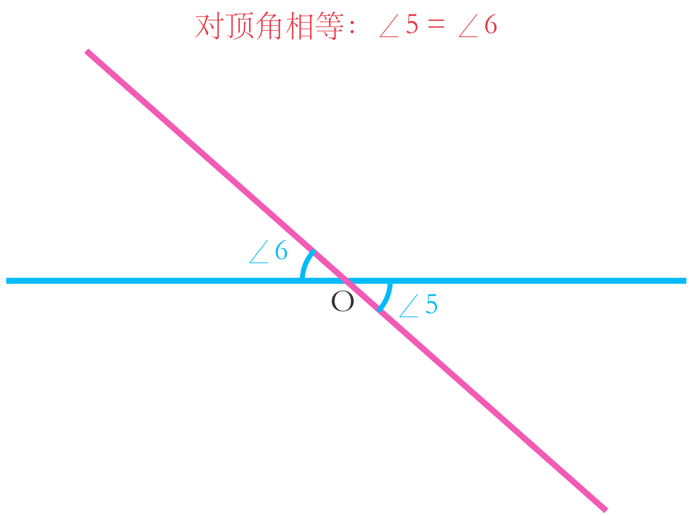
> As shown, $l_1 \parallel l_2$, AB is the transversal
> $\angle 3$ and $\angle 4$ are on the inner side of the parallel lines, called alternate interior angles
> By the parallel lines property: $\angle 3 = \angle 4$



5 Vertical Angles

Theorem: When two lines intersect, vertical angles are equal

> As shown, two lines intersect at O
> $\angle 5$ and $\angle 6$ are vertical angles
> By the vertical angles property: $\angle 5 = \angle 6$



6 Parallelogram Opposite Sides

Definition: A quadrilateral with both pairs of opposite sides parallel

Theorem: Opposite sides of a parallelogram are equal (AB = CD, AD = BC)

Proof: Connect diagonal AC

In △ABC and △CDA:

① ∠BAC = ∠DCA (AB // CD, alternate interior angles equal, §4)

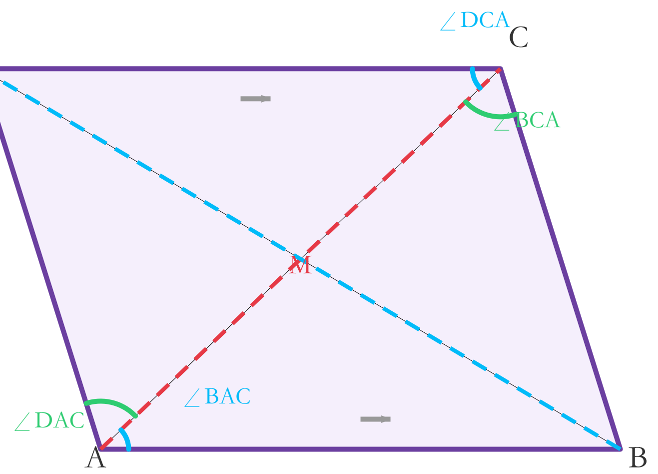
② ∠BCA = ∠DAC (AD // BC, alternate interior angles equal, §4)

③ AC = AC (common side)

∴ △ABC ≌ △CDA (ASA)

∴ AB = CD, BC = DA

This theorem will be used in the next section to prove area



7 Area of a Parallelogram

Theorem: Area of parallelogram = base × height

Proof: From D, draw DE ⊥ AB at E; from C, draw CF ⊥ AB at F

In Rt△DEA and Rt△CFB:

① AD = BC (§6 parallelogram opposite sides equal)

② ∠DAE = ∠CBF (AD // BC, §3 corresponding angles equal)

③ ∠DEA = ∠CFB = 90° (construction)

∴ △DEA ≌ △CFB (AAS)

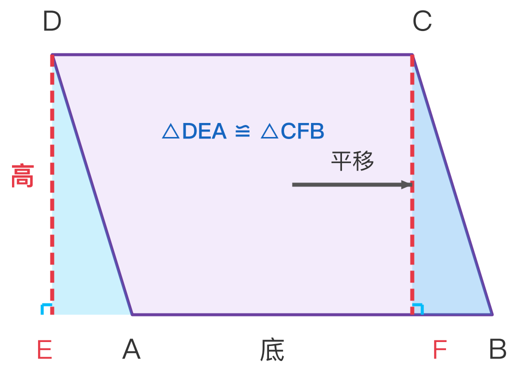
$\therefore S_{\triangle DEA} = S_{\triangle CFB}$

$S_{\square ABCD} = S_{\text{trapezoid DEAB}} + S_{\triangle DEA}$

$= S_{\text{trapezoid DEAB}} + S_{\triangle CFB}$

$= S_{\text{rectangle DEFC}}$

$= DE \times EF = \text{height} \times \text{base}$



$$\boxed{\text{Area of Parallelogram} = \text{base} \times \text{height}}$$

base: Any side of the parallelogram (e.g., AB) height: Perpendicular distance from opposite side to the base (e.g., DE)

8 Triangle Area Formula

Theorem: Area of triangle = ½ × base × height

Proof: Given △ABC, draw a line through A parallel to BC,
and a line through C parallel to AB; they intersect at D
∴ ABCD is a parallelogram (by definition)

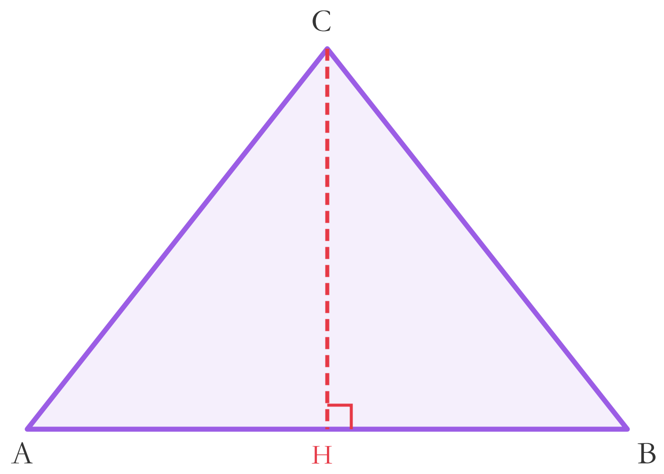
In △ABC and △CDA:
① AC = AC (common side)
② AB = CD, BC = DA (§6 parallelogram opposite sides equal)
∴ △ABC ≌ △CDA (SSS)

∴ $S_{\triangle ABC} = S_{\triangle CDA} = \frac{1}{2} \times S_{\text{▱}ABCD}$
By §7, $S_{\text{▱}ABCD} = \text{base} \times \text{height}$

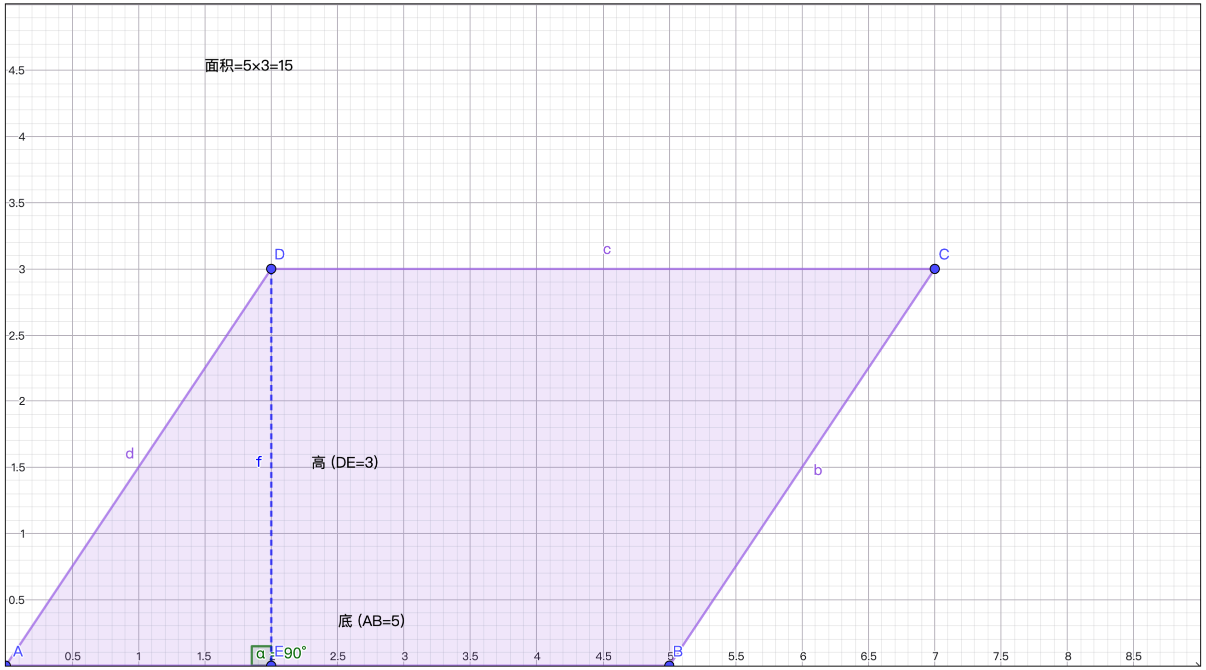
∴ $S_{\triangle ABC} = \frac{1}{2} \times \text{base} \times \text{height}$

$$S_{\triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

base: Any side of the triangle (e.g., AB) **height**: Perpendicular distance from opposite vertex to the base (e.g., CH, where H is the foot of perpendicular)



25. Area of Parallelograms and Triangles



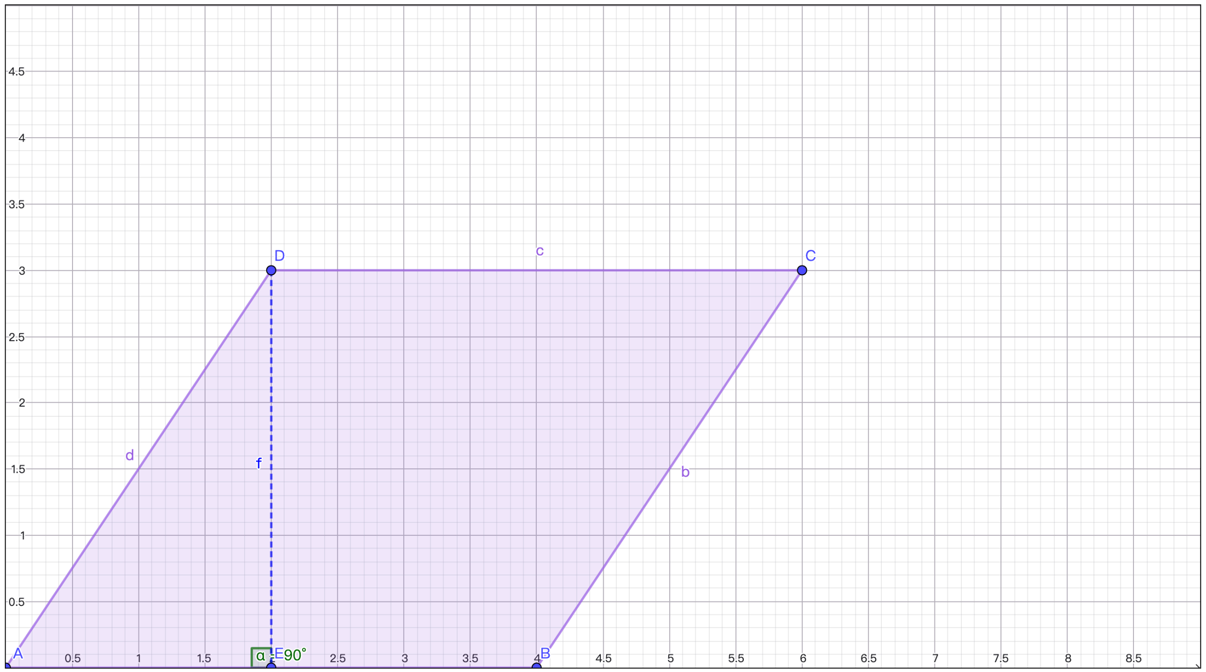
a) Demo · Area of Parallelogram

base = 5, height = 3

Area = base × height

$$= 5 \times 3 = 15$$

Area of parallelogram = base × height



b) Parallelogram

base = 4, height = 3

Area = base × height

Area =

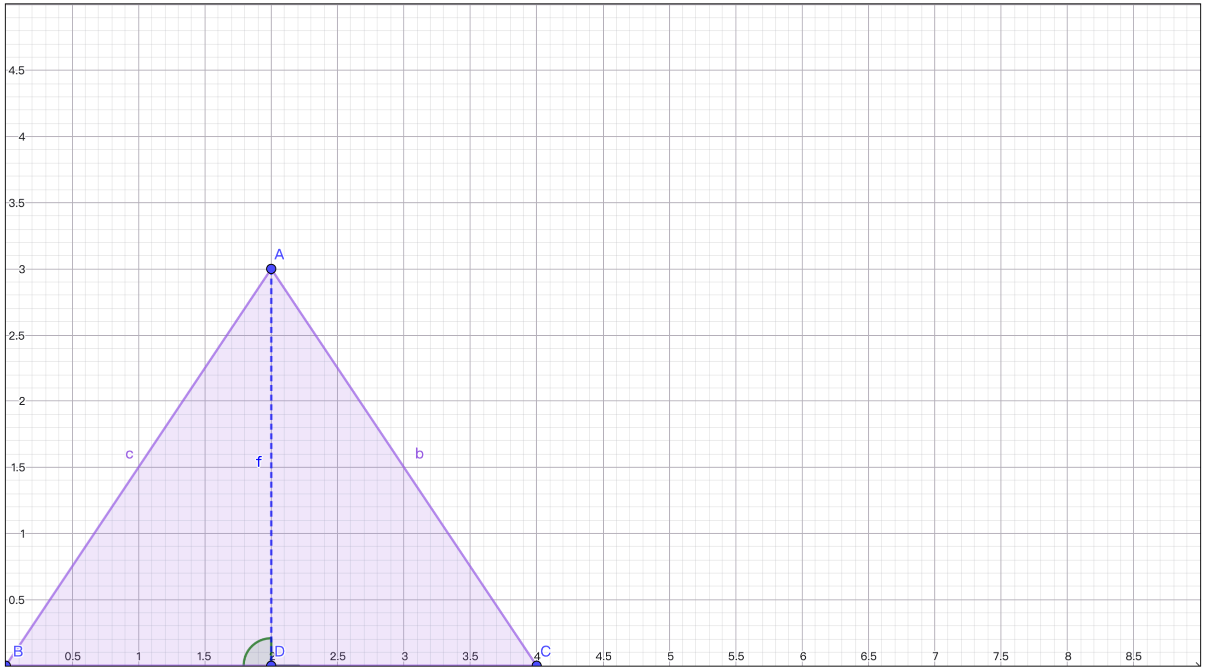
25. Area of Parallelograms and Triangles

c) Triangle

base = **4**, height = **3**

Area = **½** × **base** × **height**

Area =

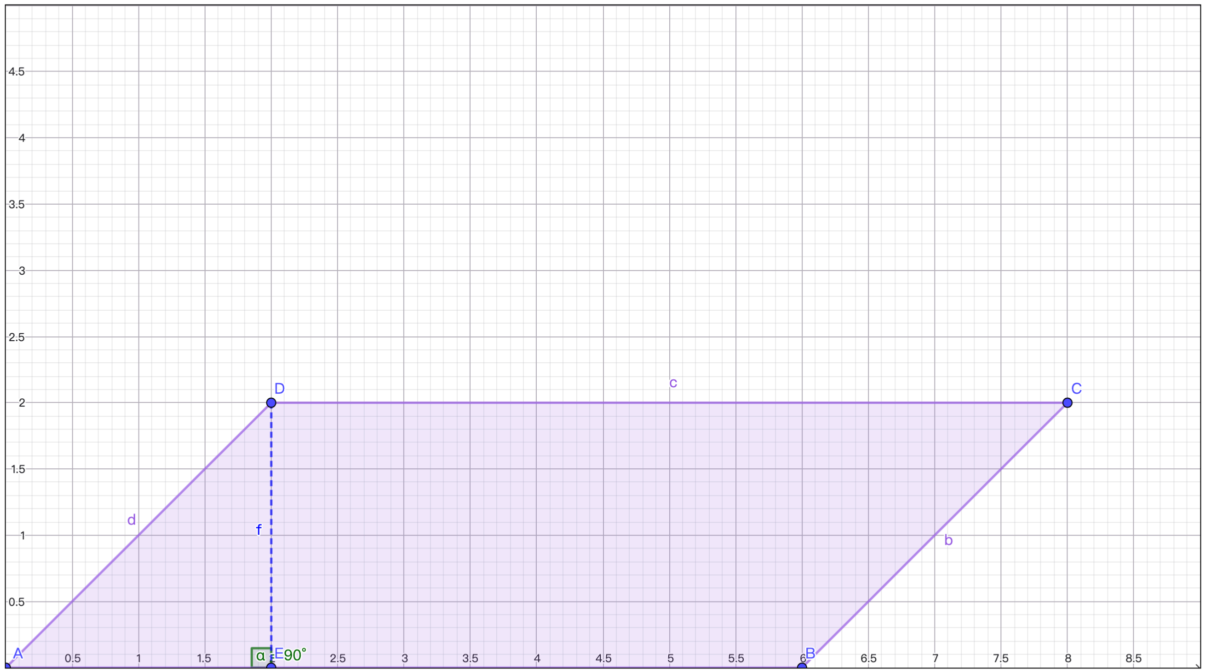


d) Parallelogram

base = **6**, height = **2**

Area = **base** × **height**

Area =



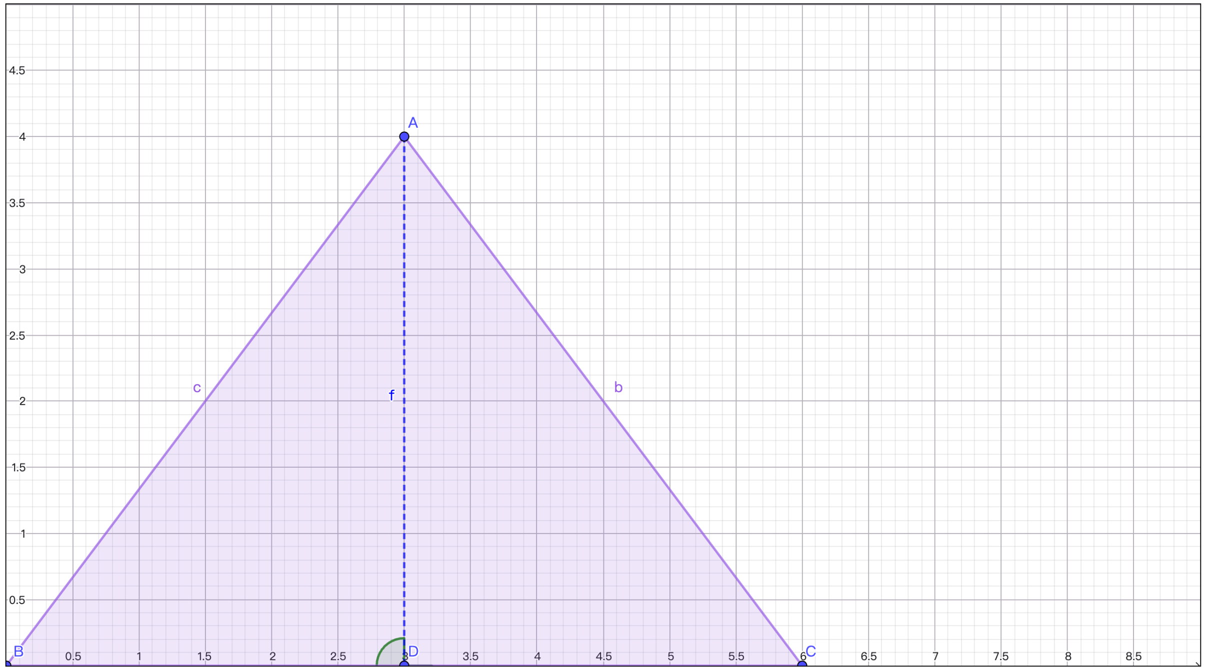
25. Area of Parallelograms and Triangles

e) Triangle

base = **6**, height = **4**

Area = **½** × **base** × **height**

Area =

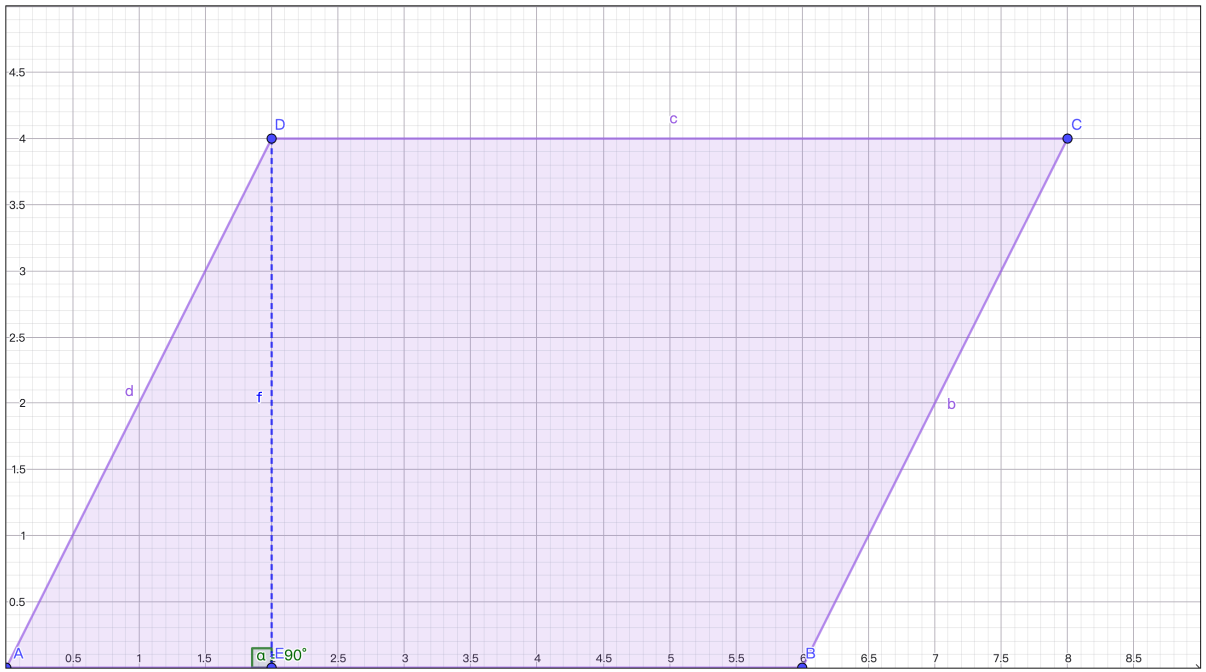


f) Parallelogram

Area = **24**, base = **6**

height = Area ÷ base

height =



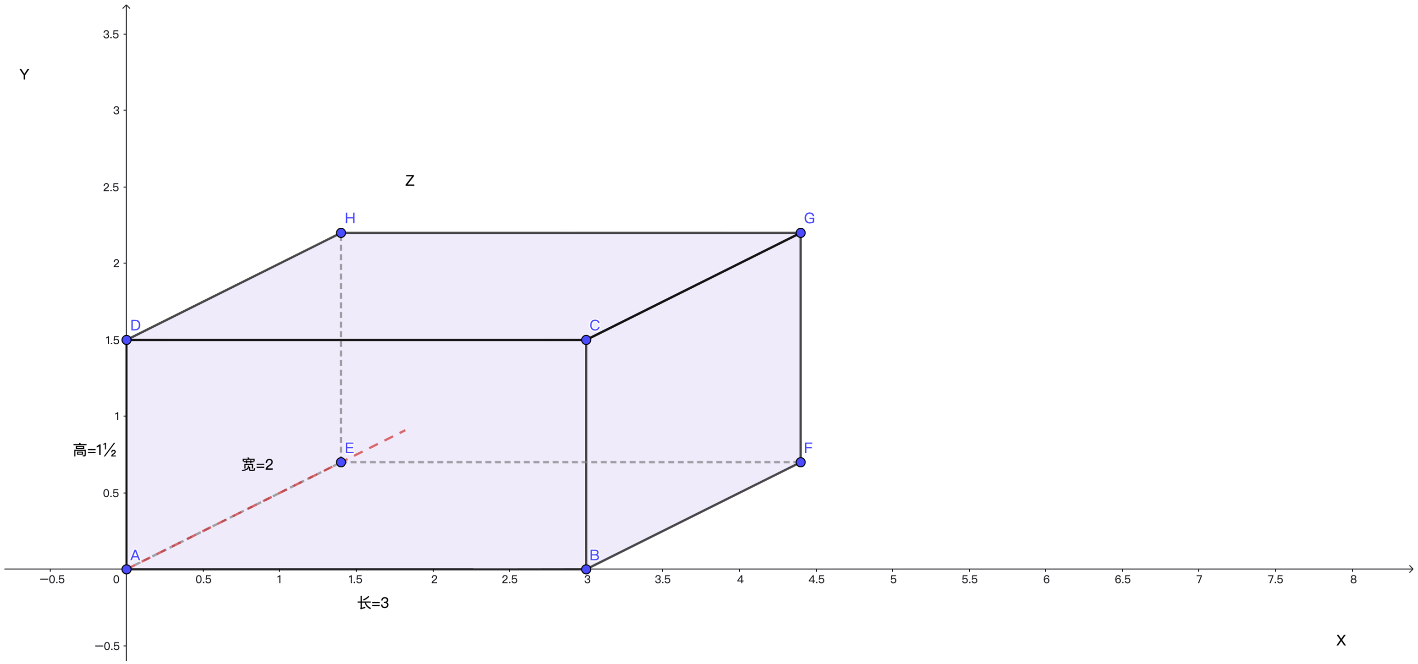
26. Volume with Fractional Edge Lengths

a) Demo · Volume of Rectangular Prism

length = 3, width = 2, height = 1½

V = length × width × height

$$= 3 \times 2 \times 3/2 = 9$$

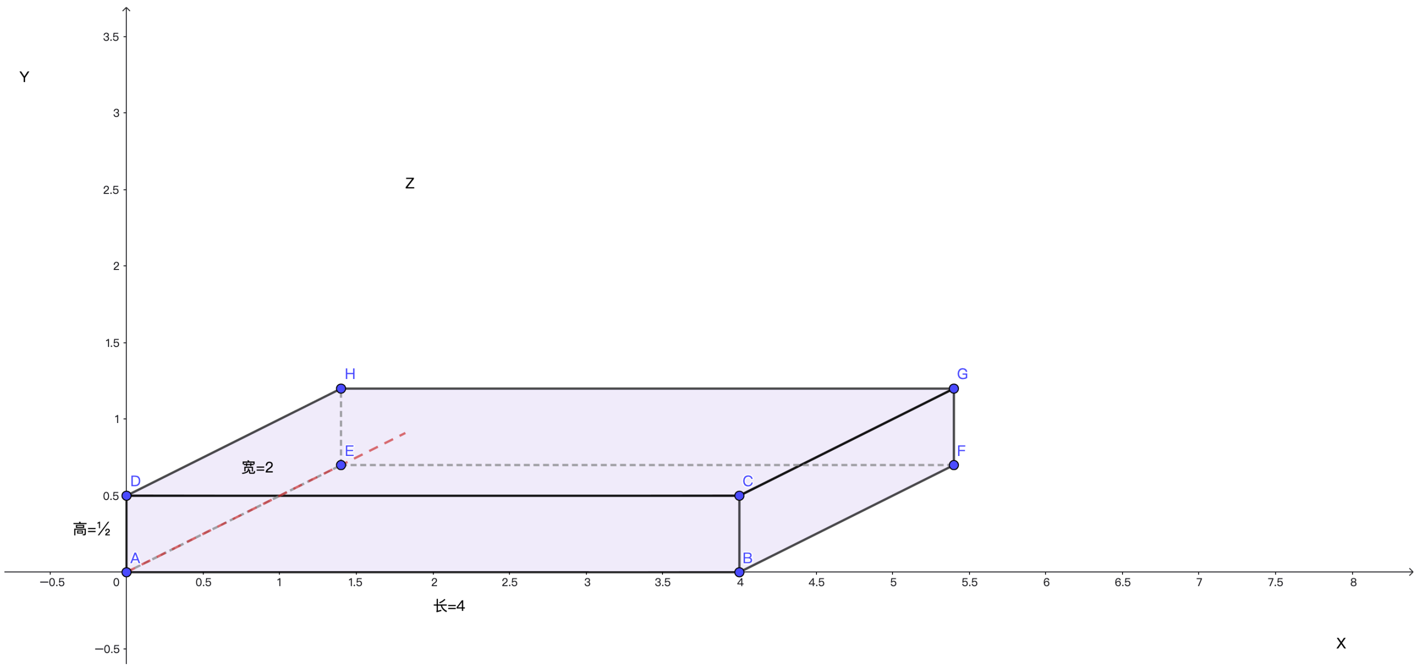


b) Rectangular Prism

length = 4, width = 2, height = ½

V = length × width × height

V =



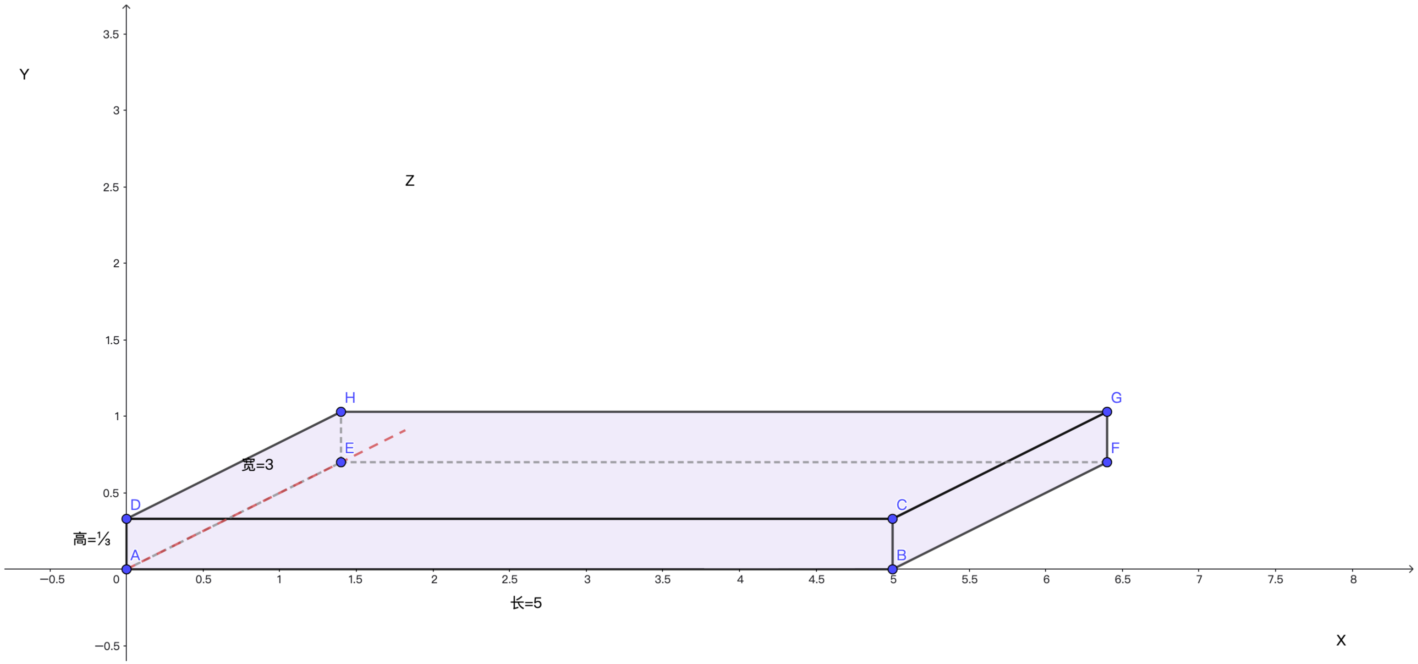
26. Volume with Fractional Edge Lengths

c) Rectangular Prism

length = 5, width = 3, height = $\frac{1}{3}$

V = length × width × height

V =

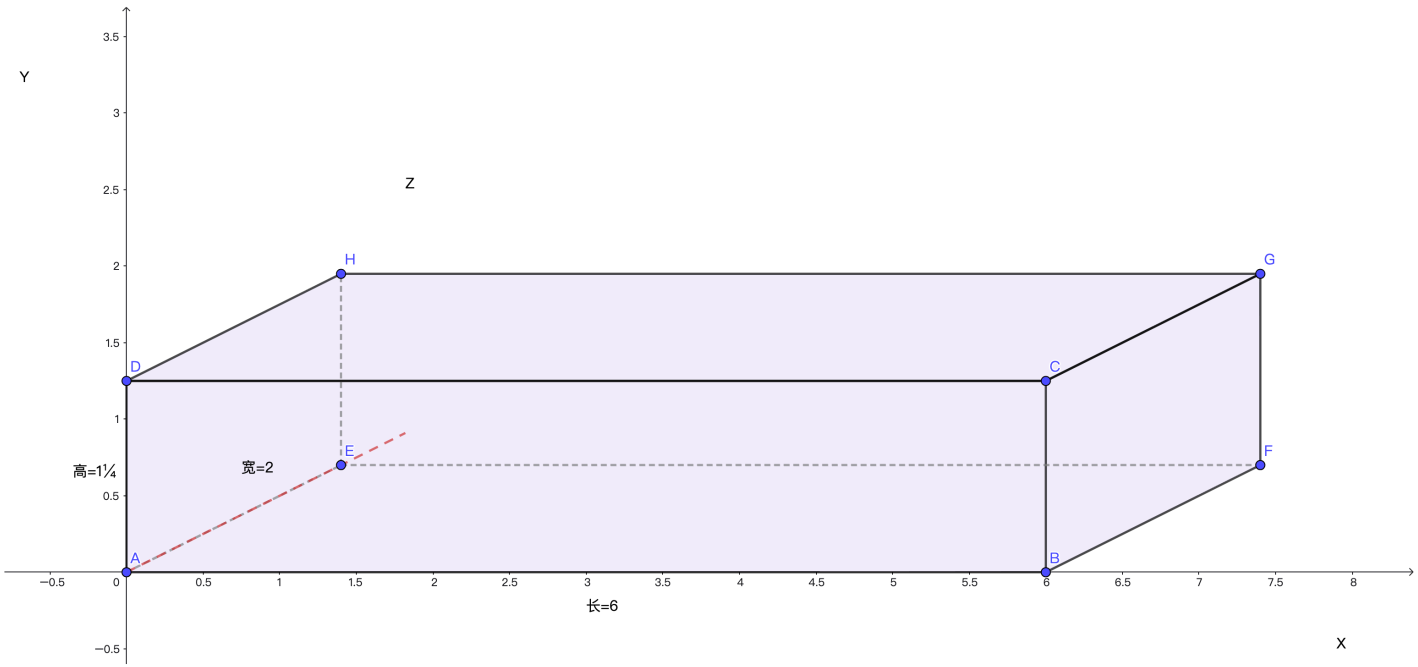


d) Rectangular Prism

length = 6, width = 2, height = $1\frac{1}{4}$

V = length × width × height

V =



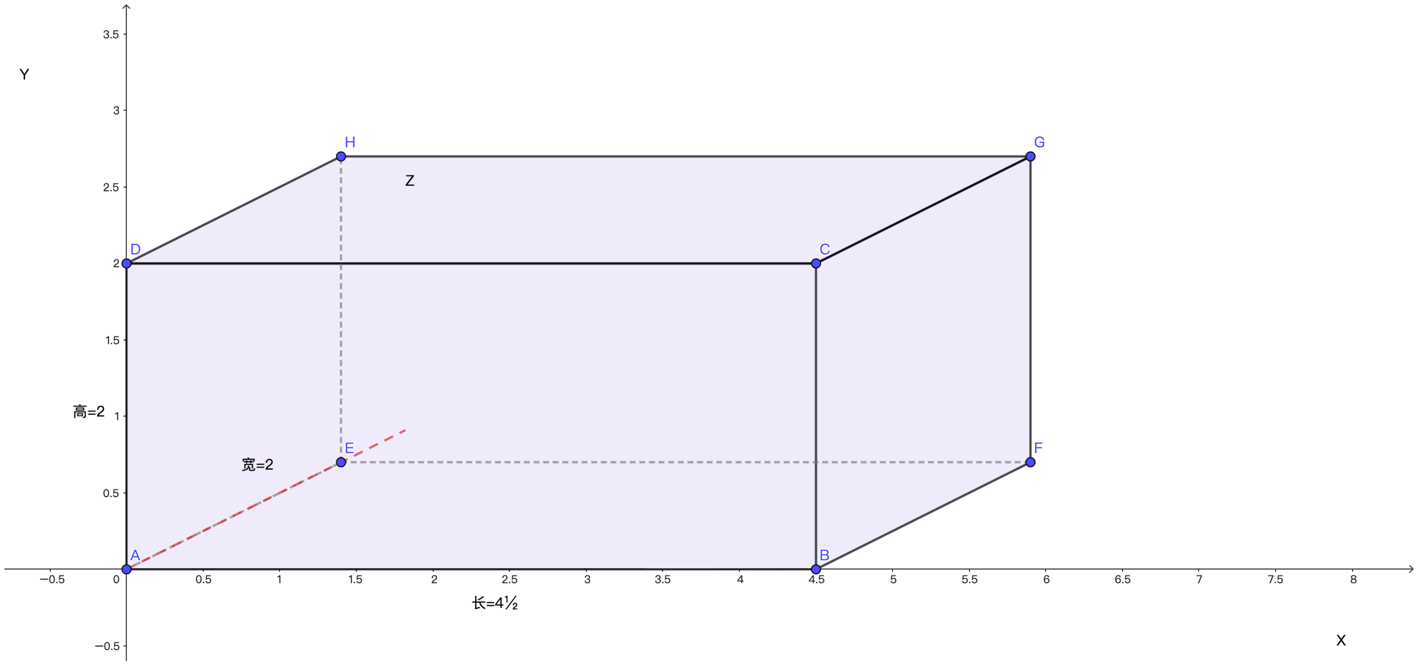
26. Volume with Fractional Edge Lengths

e) Rectangular Prism

length = 4½, width = 2, height = 2

V = length × width × height

V =

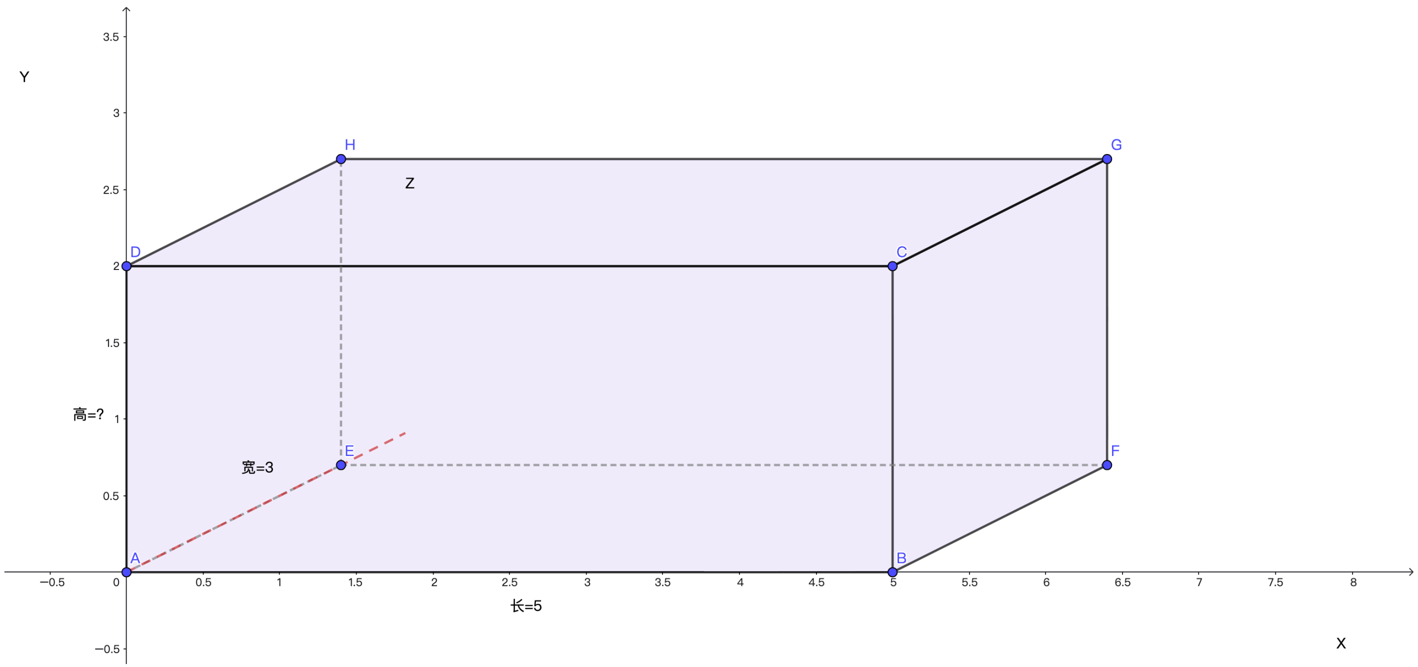


f) Rectangular Prism

V = 30, length = 5, width = 3

height = V ÷ (length × width)

height =



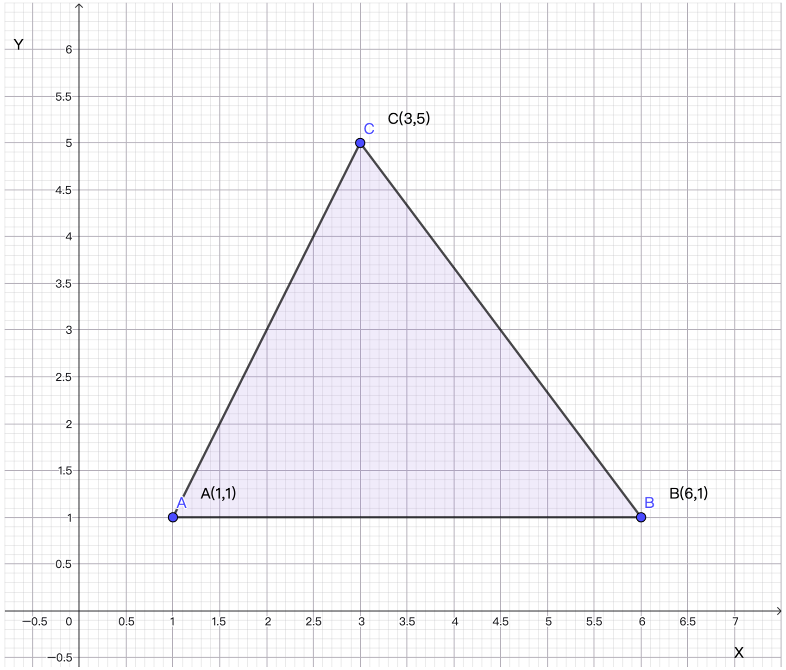
27. Polygons on the Coordinate Plane

a) Triangle

Vertices: **A**(1,1), **B**(6,1), **C**(3,5)

base = 5, height = 4

Area =

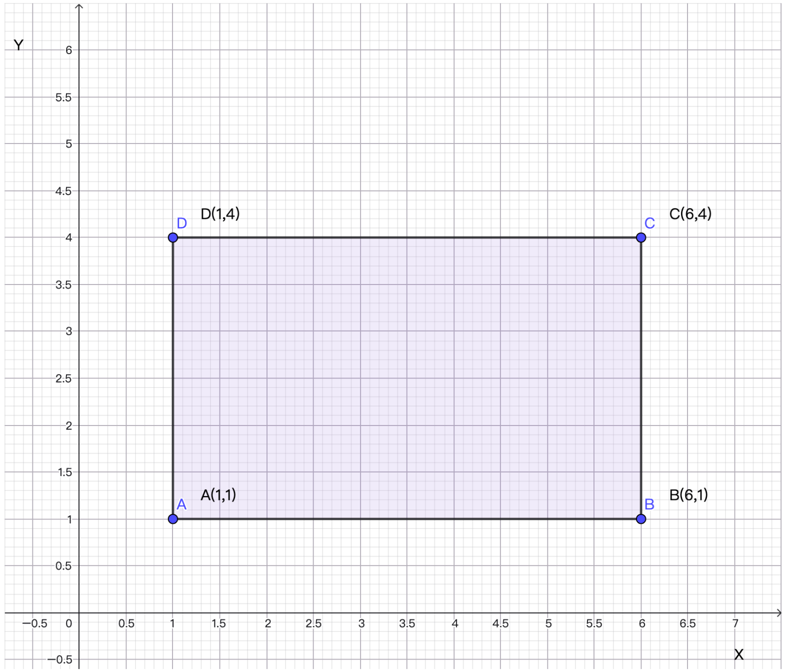


b) Rectangle

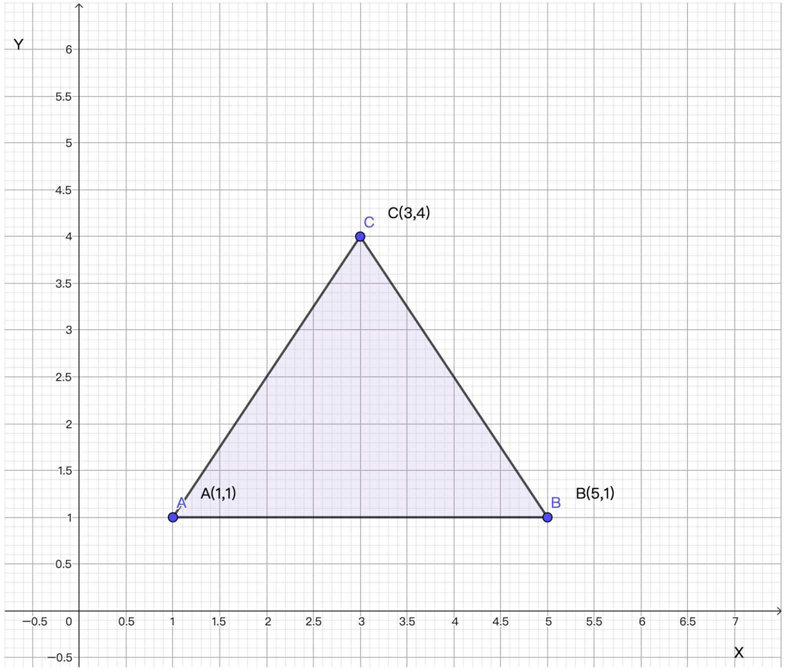
Vertices: **A**(1,1), **B**(6,1), **C**(6,4), **D**(1,4)

length = , width =

Perimeter =



27. Polygons on the Coordinate Plane

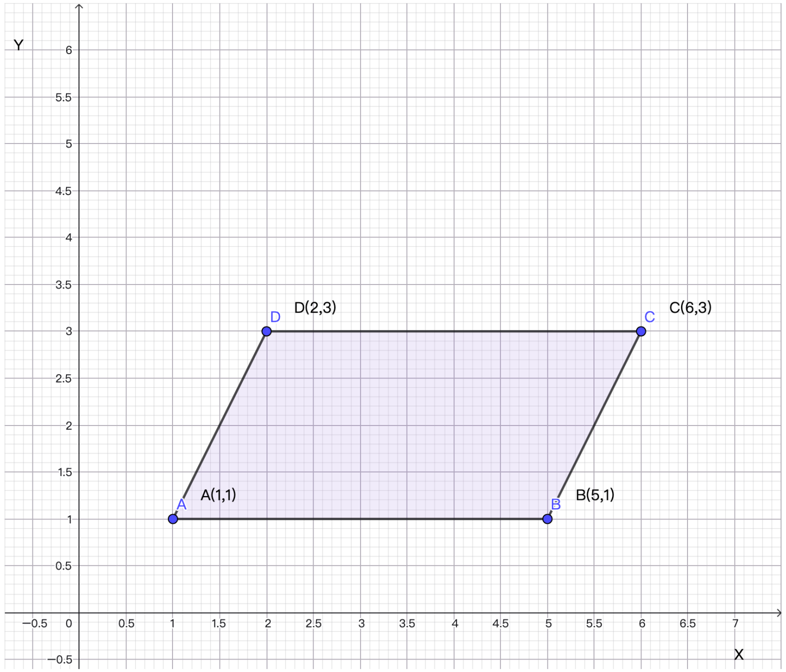


c) Triangle

Vertices: A(1,1), B(5,1), C(3,4)

base = ____, height = ____

Area = ____



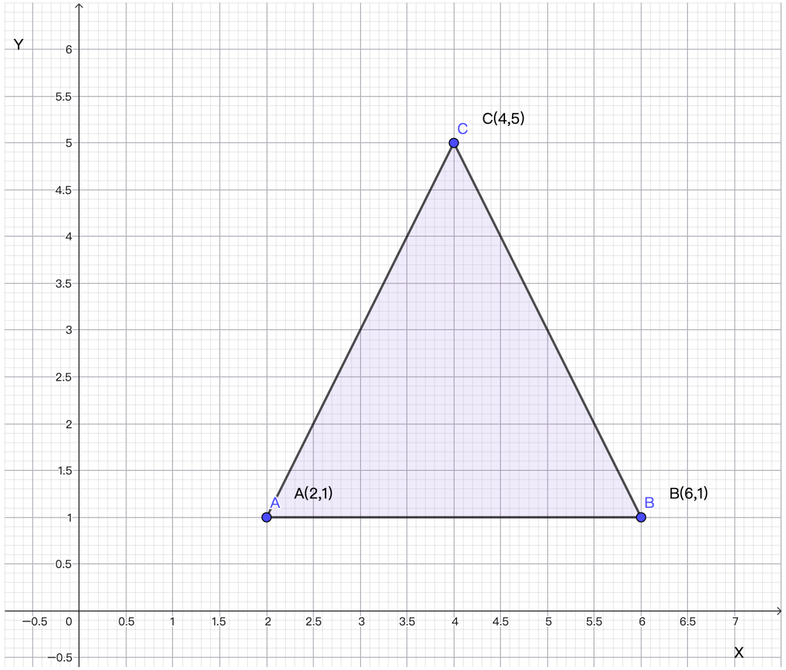
d) Parallelogram

Vertices: A(1,1), B(5,1), C(6,3), D(2,3)

base = ____, height = ____

Area = ____

27. Polygons on the Coordinate Plane

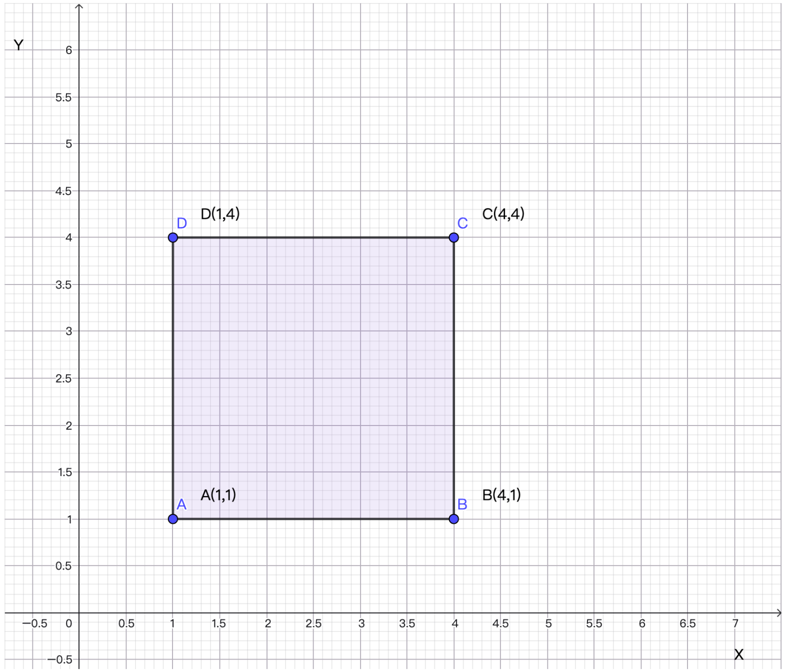


e) Triangle

Vertices: A(2,1), B(6,1), C(4,5)

base = ______, height = ______

Area = ______



f) Square

Vertices: A(1,1), B(4,1), C(4,4), D(1,4)

side length = ______

Area = ______, Perimeter = ______

28. Nets and Surface Area

a) Demo · Surface Area of Rectangular Prism

length = 3, width = 2, height = 1

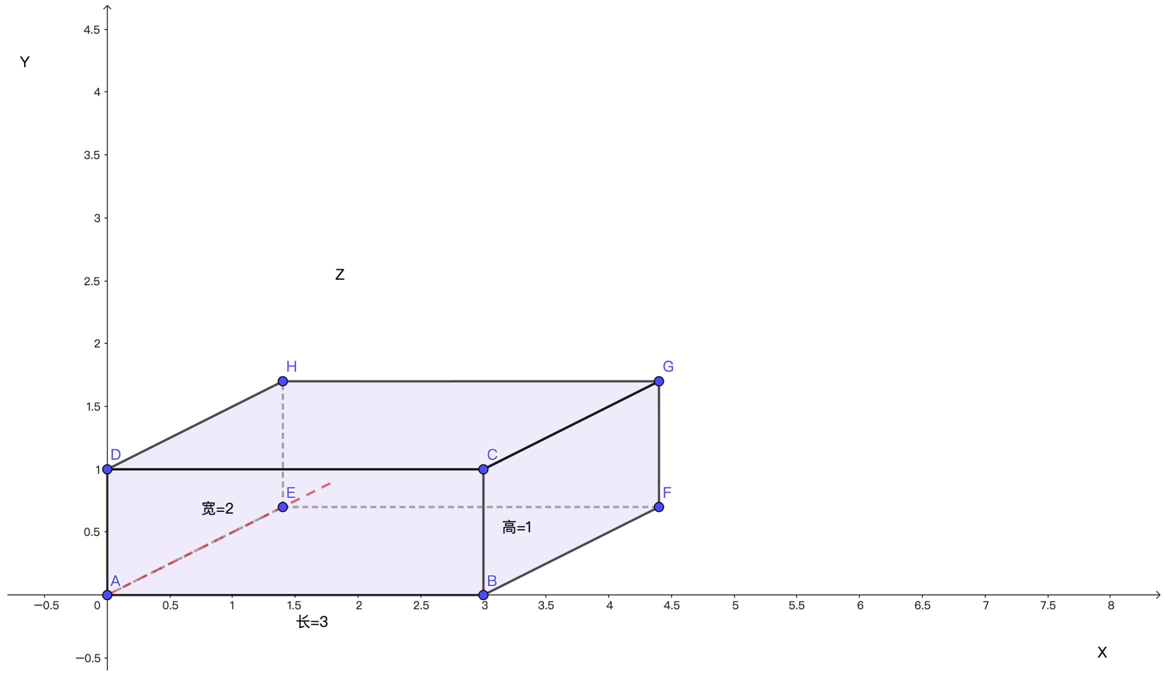
Surface Area = 2 × (length×width + length×height + width×height)

$$S = 2 \times (3\times2 + 3\times1 + 2\times1)$$

= 2 × (6 + 3 + 2)

= 2 × 11

= 22

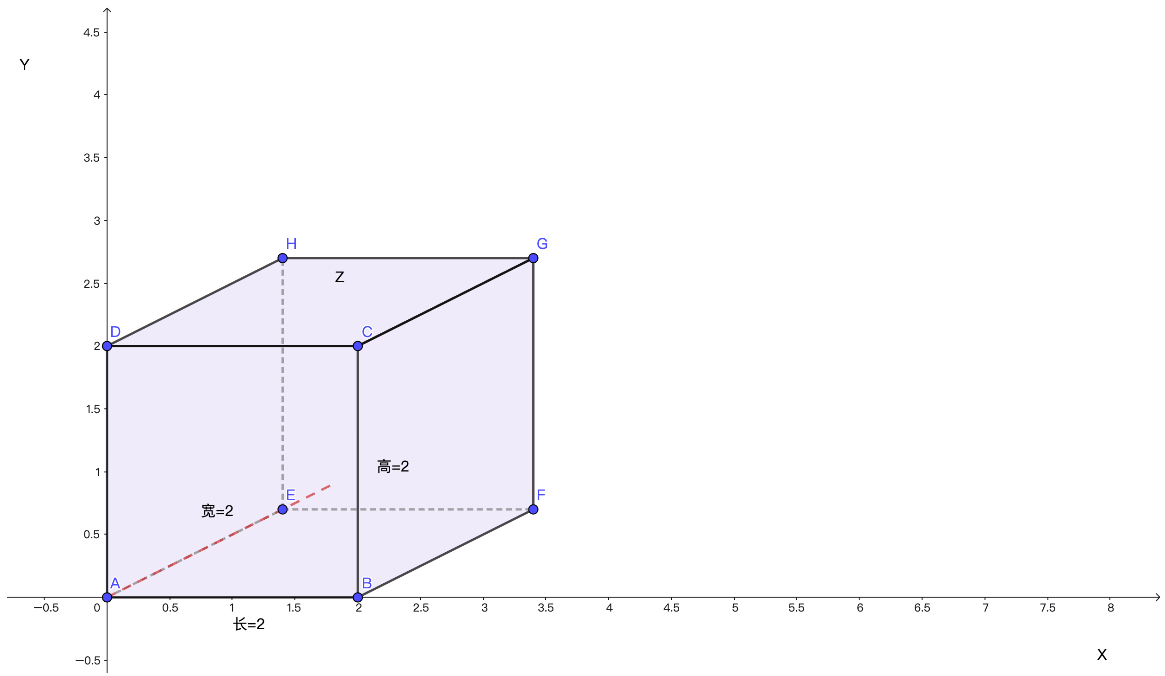


b) Cube

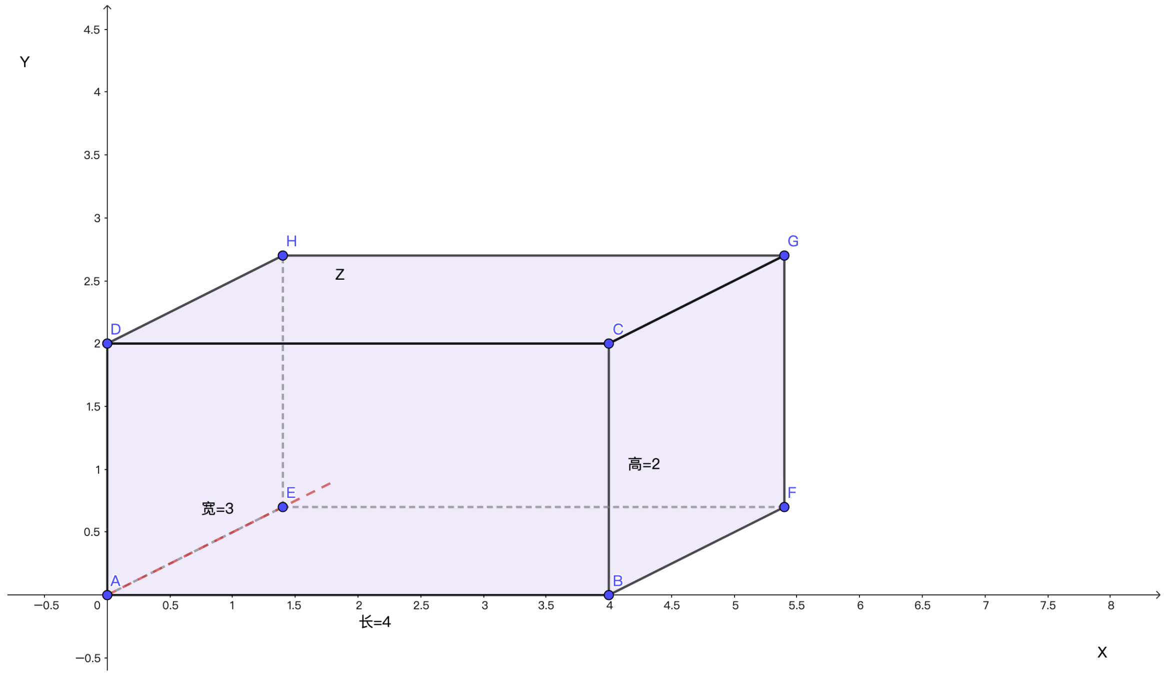
length = 2, width = 2, height = 2

Surface Area = 6 × (side×side)

Surface Area =



28. Nets and Surface Area

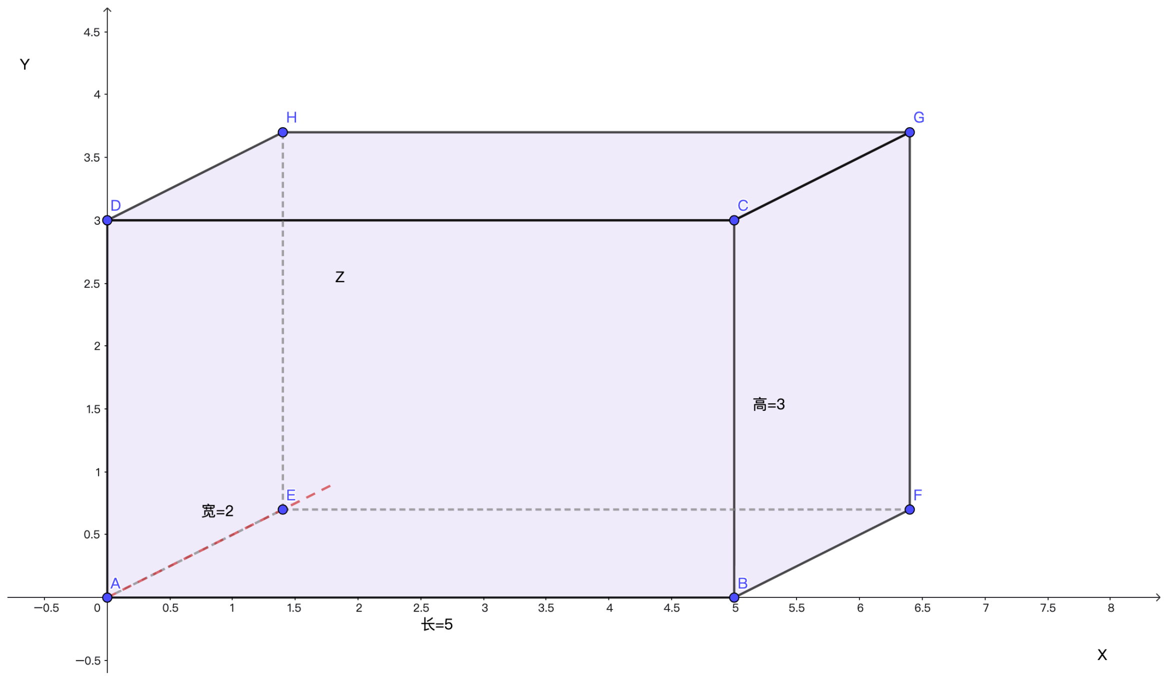


c) Rectangular Prism

length = 4, width = 3, height = 2

Surface Area = 2 × (length×width + length×height + width×height)

Surface Area =



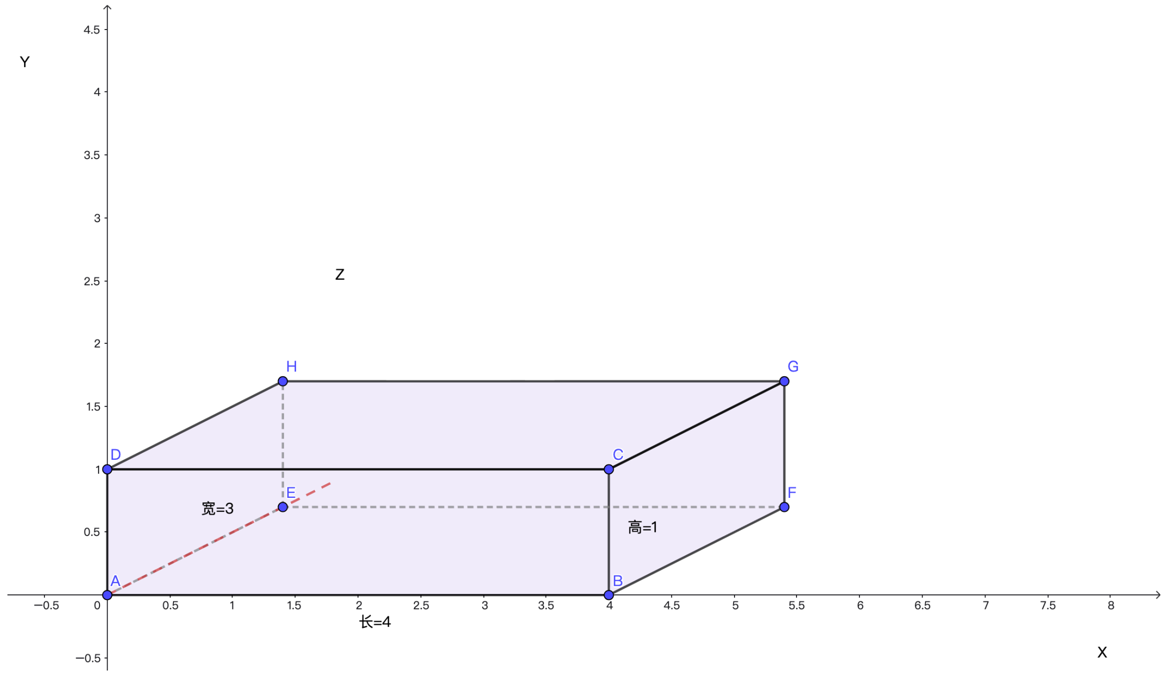
d) Rectangular Prism

length = 5, width = 2, height = 3

Surface Area = 2 × (length×width + length×height + width×height)

Surface Area =

28. Nets and Surface Area

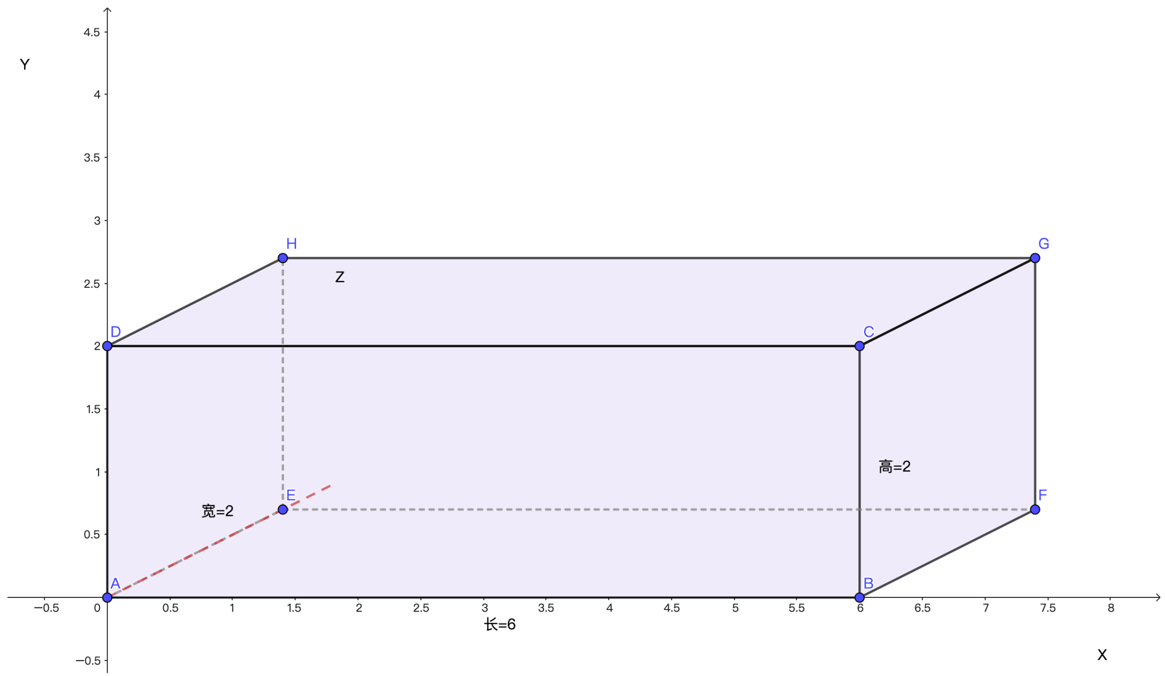


e) Rectangular Prism

Surface Area = **38**, length = **4**, width = **3**

Surface Area = 2 × (length×width + length×height + width×height)

Height =



f) Gift Box

A gift box has length **6** cm, width **2** cm, height **2** cm

How many square centimeters of wrapping paper are needed?

Wrapping paper area =

29. Statistical Questions

a) Demo

Statistical **Our class has 56 students. Over the past 30 days, what is the average sleep duration?**

Requires data: 56 students, daily sleep data for the past 30 days

Non-statistical **How many pages does my language textbook have?**

The answer is fixed — just check the last page; no data collection needed

Statistical Questions

Statistical: Data varies
different results → data collection needed

Non-statistical: Fixed answer
no data collection needed

29. Statistical Questions

b) Pencil Length

I have 10 pencils. What is the average length?

Requires data: length of each of the ______ pencils

c) Height

What is the average height of the 30 boys in our class?

Requires data: height of ______ students

29. Statistical Questions

d) Temperature

What was the daily high temperature this month?

Requires data: daily high temperatures for ______ days

e) Favorite Color

What is the favorite color of the 600 students in the school?

Requires data: responses from ______ students

f) Test Scores

What was the average math midterm score of the 45 students in the class?

Requires data: math scores of ______ students

30. Data Distribution

a) Demo · Data Distribution

Heights of 10 students (cm):

138, 142, 145, 142, 150, 138, 145, 148, 142, 145

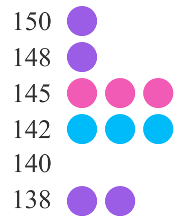
Minimum: 138   **Maximum**: 150

Range: 150 − 138 = **12**

Most frequent value(s) (mode): 142 and 145

Distribution shape: More in the middle, fewer at the ends

Height Dot Plot



b) Dot Plot

Test scores of 8 students:

85, 90, 85, 95, 85, 90, 88, 92

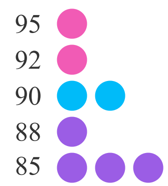
① Minimum = ____

② Maximum = ____

③ Range = ____

④ Most frequent score = ____

Score Dot Plot



30. Data Distribution

c) Reading Dot Plots

The dot plot below shows the **weekly reading time (hours)** of 12 students:

① How many data points were collected?

___ data points

② What is the range of reading time?

From ___ to ___ hours

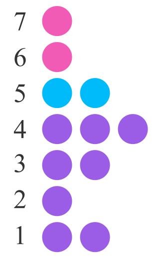
③ Most frequent reading time?

___ hours

④ How many read more than 4 hours?

___ students

Reading Time Dot Plot



d) Comparing Data Distributions

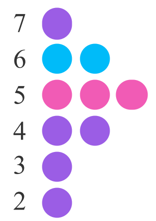
Examine the dot plots for datasets A, B, and C below:

Set A: **2, 3, 4, 4, 5, 5, 5, 6, 6, 7**

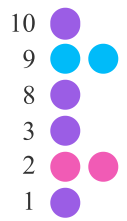
Set B: **1, 2, 2, 3, 8, 9, 9, 10**

Set C: **1, 1, 1, 1, 1, 9, 9, 9, 9, 9**

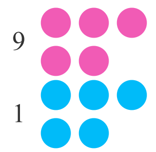
Set A



Set B



Set C



30. Data Distribution

e) Drawing Dot Plots

Draw a dot plot for the following data on the right:

3, 5, 4, 6, 3, 5, 5, 7, 4, 5, 6, 3

① Data range: from ____ to ____

② Which value appears most? ____

Your Dot Plot Here

f) Interpreting Data

The dot plot below shows **weekly TV watching time (hours)** of a class:

① What is the most common TV watching time?

____ hours

② How many hours does the student who watches the most spend on TV?

____ hours

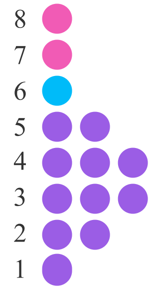
③ If you watch 5 hours per week, is that more or less than most students?

____

④ What are the characteristics of this distribution?

____

TV Watching Time Dot Plot



31. Measures of Center and Variability

a) Demo · Measures of Center

Mean	**Sum ÷ Count**
Median	**Middle value when sorted**
Mode	**Most frequent value**
Range	**From minimum to maximum**

Test scores of 7 students:

72, 85, 85, 90, 92, 95, 100

Sorted: 72, 85, 85, **90**, 92, 95, 100

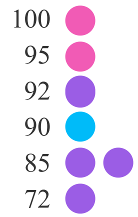
Mean = (72+85+85+90+92+95+100) ÷ 7 ≈ **88.4**

Median = 4th value = **90** (3 on each side)

Mode = **85** (appears 2 times)

Range: from 72 to 100, span = 100 − 72 = **28**

Score Dot Plot



b) Finding the Mean

Jump rope counts of 6 students (jumps/min):

45, 52, 48, 55, 50, 44

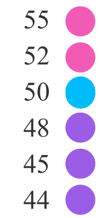
① Sum =

② Count =

③ Mean ≈

④ Looking at the dot plot, what range do most students fall into?

Jump Rope Dot Plot



31. Measures of Center and Variability

c) Finding the Median

Median: When sorted, the number with equal counts on both sides. If even count, average the two middle values.

① For 7 values, the 4th is the median

3, 3, 5, 5, 5, 5, 5 (five 5's, two 3's)

Sorted: 3, 3, 5, **5**, 5, 5, 5

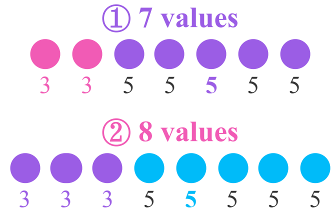
Median = ___ (4th value)

② For 8 values, average of 4th and 5th

3, 3, 3, 5, 5, 5, 5, 5 (five 5's, three 3's)

Sorted: 3, 3, 3, **5**, **5**, 5, 5, 5

Median = (___ + ___)÷2 = ___



d) Mode and Range

Mode: The value that appears most often. A dataset may have one mode, multiple modes, or no mode.

① **4, 4, 4, 6, 7, 7, 9**

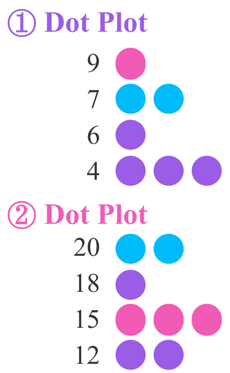
Mode = ___ Range = ___

② **12, 12, 15, 15, 15, 18, 20, 20**

Mode = ___ Range = ___

③ **23, 25, 25, 28, 28, 30**

Mode = ___ Range = ___



31. Measures of Center and Variability

e) Choosing the Right Measure

Weekly allowance of 10 students (yuan):

20, 20, 20, 20, 25, 25, 25, 30, 30, 100

① Mean ≈

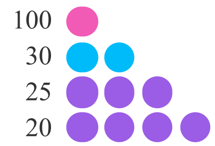
② Median =

③ Mode =

④ Which is most affected by the outlier (100 yuan)?

⑤ Which measure best represents most students?

Allowance Dot Plot



100 is an outlier that pulls up the mean

f) Comparing Two Datasets

Math test scores of 10 students from Class A and Class B:

Class A: **75, 78, 80, 82, 85, 85, 88, 90, 92, 95**

Class B: **60, 65, 70, 75, 85, 85, 90, 95, 98, 100**

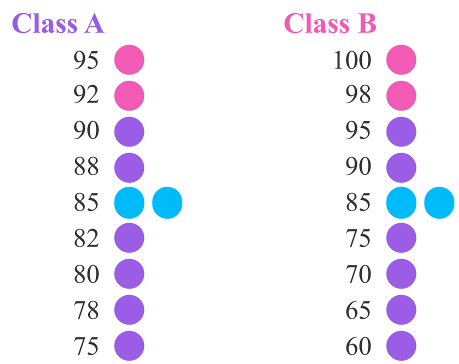
① Mean of A ≈

Mean of B ≈

② Range of A =

Range of B =

③ Which class has more consistent scores? Why?



32. Data Displays

a) Demo · Dot Plot

Number of pets owned by 12 students:

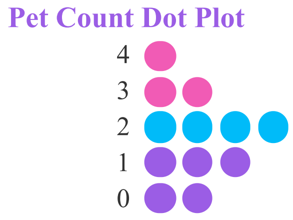
0, 0, 1, 1, 1, 2, 2, 2, 2, 3, 3, 4

Each dot represents one student:

• Most students have **2** pets

• Data range: from **0** to **4**

• **1** student has more than 3 pets



b) Demo · Histogram

Daily sleep time of 30 students grouped:

A histogram groups data; taller bar means more people:

• Which group has the most? **8~9 hours**

• **4** students sleep less than 7 hours

• **7** students sleep at least 9 hours

32. Data Displays

c) Reading Dot Plots

The dot plot shows **daily exercise time (minutes)** of 30 students:

① How many data points?

② What is the exercise time range?

From   to   minutes

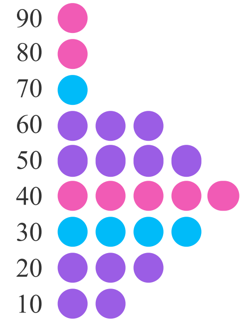
③ What is the most common exercise time?

  minutes

④ How many exercise more than 60 minutes?

  students

Exercise Time Dot Plot



d) Reading Histograms

The histogram shows **heights (cm)** of 40 students:

① Which height range has the most students?

② How many are 140~149cm?

  students

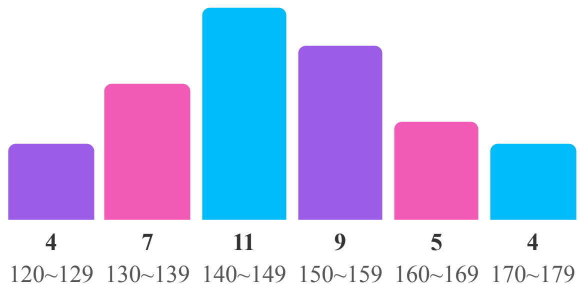
③ How many are at least 160cm?

  students

④ How many are under 150cm?

  students

Height Histogram



32. Data Displays

e) Reading Dot Plots

The dot plot shows **weekly allowance (yuan)** of 15 students:

① What is the most common allowance?

yuan

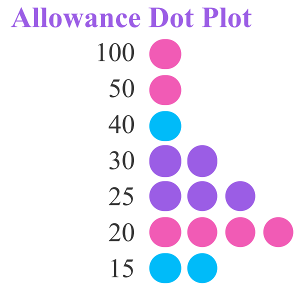
② What is the highest allowance?

yuan

③ Any unusually high values?

④ How many have less than 30 yuan?

students



f) Reading Histograms

The histogram shows **math test scores** of 50 students:

① Which score range has the most?

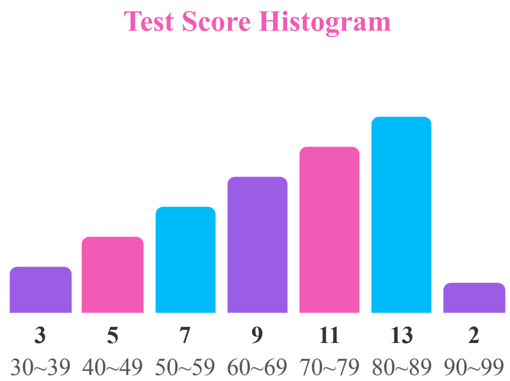
② How many passed (above 60)?

students

③ How many scored above 80?

students

④ Which score range has the fewest?



g) Comprehensive Application

The dot plot shows **weekly TV watching time (hours)** of 25 students:

① What is the data range?

From ___ to ___ hours

② What is the most common watching time?

___ hours

③ What is the longest watching time?

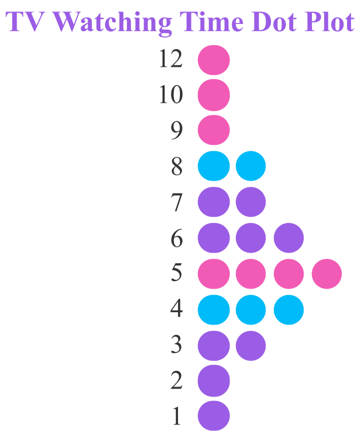
___ hours

④ How many watch more than 5 hours?

___ students

⑤ What are the characteristics of this distribution?

___



33. Summarizing Data

a) Demo · Summarizing Data

Range	**From minimum to maximum**
Mean	**Sum ÷ Count**
Median	**Middle value when sorted**
Mode	**Most frequent value**

Daily sleep time of 15 students (hours):

7, 7.5, 8, 8, 8, 8.5, 8.5, 8.5, 9, 9, 9, 9.5, 9.5, 10, 10

• Range: from 7 to 10, span of 3 hours

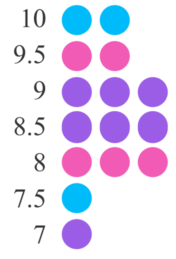
• Mean ≈ **8.6** hours

• Median = 8th value = **8.5** hours

• Mode = **8, 8.5, 9** (each appears 3 times)

Conclusion: Most students sleep 8~9 hours

Sleep Time Dot Plot



b) Practice Summarizing Data

Daily phone usage of 12 students (minutes):

30, 45, 45, 60, 60, 60, 75, 75, 90, 90, 120, 150

① Minimum = ____ Maximum = ____

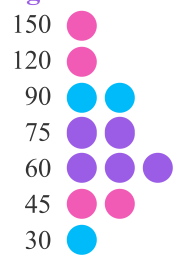
② Range: from ____ to ____

③ Mean ≈ ____

④ Median = ____

⑤ Mode = ____

Phone Usage Dot Plot



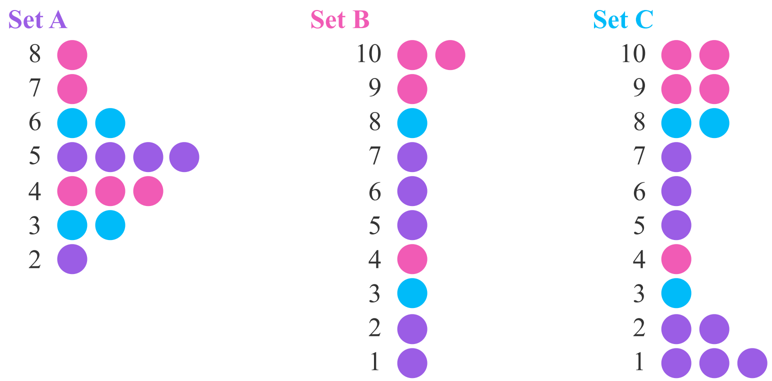
c) Describing Distribution Shapes

Observe the three dot plots and describe their characteristics:

① Shape of Set A:

② Shape of Set B:

③ Shape of Set C:



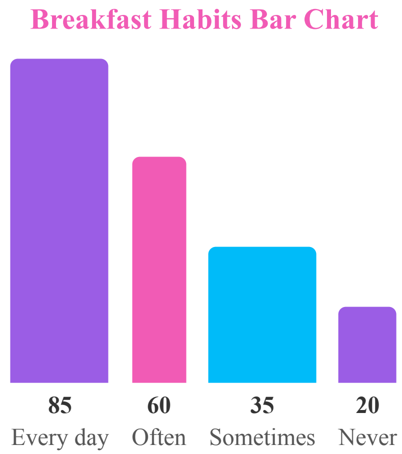
d) Real-World Interpretation

The bar chart shows **daily breakfast habits** of 200 students:

① What describes most students' breakfast habits?

② What are the characteristics of this distribution?

③ Based on the data, what could the school do to improve?



33. Summarizing Data

e) Comparing Two Datasets

Math scores of 12 students from Class 3 and Class 4:

Class 3: **65,70,72,75,78,80,82,85,88,90,92,95**

Class 4: **50,55,60,65,70,75,80,85,90,95,98,100**

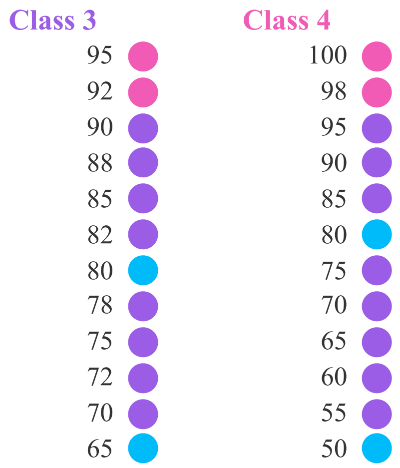
① Mean of Class 3 ≈

Mean of Class 4 ≈

② Range of Class 3 =

Range of Class 4 =

③ Which class has more concentrated scores?



f) Writing a Data Summary

Weekly exercise time of 10 students (minutes):

90, 120, 120, 150, 150, 150, 180, 180, 210, 300

① Range: from to

② Mean ≈

③ Median =

④ Mode =

⑤ Any outliers?

⑥ Write a paragraph summarizing exercise time: